Continuous concrete bridges

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Second Edition

PORTLAND CEMENT ASSOCIATION

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Continuous Concrete Bridges

Section I—Introduction

Continuous concrete bridges in units of three, four or five spans, with concrete pile or frame bents for piers are adaptable to most stream crossings and grade separations. For long spans, (probably up to 150 ft.), the continuous T-girder offers an economical solution and continuous slab bridges present advantages for spans under 35 ft. For spans exceeding the range for solid web T-girders, the continuous hollow girder concrete bridge will be most suitable and economical. Discussion in this booklet is confined to T-girder and slab bridges, but the subject of hollow girder bridges is fully treated in *Continuous Hollow Girder Concrete Bridges**.

Since continuous girder bridges are best proportioned when the interior spans are from 1.3 to 1.4 times the length of the end spans for loadings and unit stresses in common use, this type of bridge is generally more satisfactory than one consisting of simple spans because the piers can often be placed on the stream bank or outside the main channel for stream crossings and at the sides of the roadway for grade separations.

Only single bearings are required at interior supports of continuous bridges, materially reducing the width of piers compared to simple spans. Cost of the substructure is thereby reduced; obstruction to stream flow is minimized and the tendency to scour resulting from large piers is lessened.

Continuous bridges require fewer expansion joints, which reduces first cost slightly and the expense of maintaining joints.

In a well designed continuous girder bridge, the depth of sections follows closely the moment requirements, varying from a minimum at the center of spans to a maximum at the supports. The effect of dead load on the design is reduced accordingly. The variation of section from center of spans to supports is also favorable to the stress requirements.

Reduction in deck depth, particularly at mid-span, gives the continuous bridge an important economic and architectural advantage since deck type structures can often be used where through bridges, generally lacking architectural grace, were required heretofore to provide adequate roadway clearance or waterway. Future widening is facilitated with economy which is of importance with increasing demand for wider roadways.

The longer interior spans, desirable for structural reasons, and the haunched soffits of continuous bridges contribute to the good appearance of such bridges. When desirable for the sake of appearance, the depth over supports may be increased over that required at little or no increase in cost because a comparable decrease can be made in mid-span depth.

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*Available free in United States and Canada upon request to Portland Cement Association.

Continuous slab bridge on pile bents over Lafayette River, Norfolk, Va., designed by Norfolk Department of Public Works, Division of Bridges; Norman Z. Ball, principal assistant engineer in charge of design.

Section II—Layout

Most important steps in bridge design are making the layout and selecting the type of bridge to be used. Here all major economies are made or lost*. Since the general principles of these phases of bridge design are well discussed in available texts and bulletins**, the discussion here will be limited to considerations of special importance in continuous concrete bridge design.

Relationship between Span Lengths

When the over-all length of a bridge is such that it may be made in one continuous unit, the number of spans and their relative lengths are influenced by the physical conditions of the site which may fix the positions of piers and abutments or may allow sufficient freedom to secure the best arrangement for service and economy. Pier and abutment location may be fixed by highways, streets, or railroad crossings at ends of the proposed bridge or these physical conditions and the location of the main channel of the stream may dictate the location of piers between the abutments.

At sites where piers and abutments can be placed as desired, for maximum economy the ratio of intermediate span to end span within a unit should be as follows:

For slab spans—end spans up to 35 ft., 1.26
—end spans 35 to 50 ft.***, 1.31
For girder spans—end spans 35 ft. and up, 1.37 to 1.40.

**Economics of Highway Bridge Types* by C. B. McCullough, Gillette Pub. Co., Chicago.

**For comprehensive discussions of the many considerations involved in making a good layout of a bridge see: *Highway Bridge Location*, Bulletin 1486, and *Highway Bridge Surveys*, Bulletin 55; both by the U. S. Department of Agriculture and obtainable from the Superintendent of Documents, Washington, D. C.

***Generally, if span lengths in excess of 35 ft. are required it will be economical to use girder construction rather than a solid slab.

The foregoing span ratios result from the ratios of dead to live load that obtain with A.A.S.H.O. loadings and design stresses of $f_s = 18{,}000$ and $f_c = 0.40f'_c$ for positive moment and $f_c = 0.45f'_c$ for negative moment assuming a 3,000-lb. concrete. For any other loads and stresses a slight deviation from these span ratios may be expected.

The span ratios given here are for continuous decks which are not built integral with the supports. Spans of these ratios will give moments requiring the same depth of sections at centerlines of all spans and the same amount of steel, thus providing a balanced design at minimum cost for the deck. When decks are built integral with the supports the span ratios can be increased somewhat; this increase will depend upon the stiffness of supports.

In long bridges across overflow areas, long grade separations, or elevated highways, a free choice of span length will usually permit a selection to give balanced sections. This is desirable for practical arrangement of steel and also for good appearance. For girder bridges under average conditions, the economical length of end spans of groups of continuous spans not integral with supports is *approximately* as follows for various types of reinforced concrete substructures:

On pile bents—50 ft.

On open-framed bents—50 to 65 ft.

On solid piers of light construction (20 to 24 in. under cap)—60 to 80 ft.

On solid piers of heavy construction (24 to 36 in. under cap)—80 ft. and over.

Economical intermediate span lengths are obtained by applying the foregoing span length ratios to the end spans given above.

Decks Integral with Substructure

There are some advantages in building the deck and substructure of a bridge integral and there are some disadvantages which should be considered when making a layout. Among the advantages are:

1. Slightly increased ratio of interior to exterior spans may be helpful where a few extra feet are needed for clear waterway or roadway in case of a grade separation.
2. Reduced mid-span moments and increased haunch moments due to deck loads minimize required section at mid-span with consequent reduction in dead load.
3. Reduced deck depth makes reduction of grade possible.
4. Pier width may be less when made integral at top, thus widening clear waterway or making a shorter bridge possible for same waterway.
5. Stability of structure is enhanced which may be important in streams carrying large amounts of drift during floods.
6. Rollers or other bearing devices are obviated.
7. Appearance is improved.

Among the disadvantages is:

1. Temperature and shrinkage stresses must be taken into account and they may become of important magnitude where piers are short and stiff and spans are long. When the distance between first and last fixed supports exceeds 200 ft. and the height to width ratio of piers is less than about 12, volume change stresses are likely to be high.

While the advantages of making the deck and piers integral would seem to outweigh the disadvantages, the increase in stresses due to temperature changes and shrinkage in a given case may make the integral structure uneconomical. It is advisable to make comparative designs to the extent of determining critical dimensions in order to be sure the type selected is the most economical.

Hinged Spans

When a layout requires two or more groups of spans, the groups may be joined by a hinge near the point of contraflexure of one of the end spans of a group, or there may be a hinge near both points of contraflexure. Certain advantages result:

(a) The two end spans of contiguous groups may be made about as long as the intermediate spans, improving appearance of the bridge.

(b) Provision for expansion is made in the span and not at the pier, which makes a gutter take-off unnecessary.

If an expansion device is placed at each point of contraflexure, the labor involved in stress calculation is not increased. If an expansion device is placed at only one point of contraflexure, however, the stress analysis is then based on a continuous unit of a number of spans equal to the sum of the spans so connected, and the span in which the joints occur will have beam constants materially different from spans that are similar otherwise*. This will result in some increase in designing time but the saving in construction cost will usually warrant the extra labor.

Selection of Type of Substructure

Before an economical layout can be made the type of substructure must be selected because the number and length of spans will depend upon the cost of the substructure. The usual types of substructures and the conditions under which they are generally used are briefly described on the following page. A more complete discussion is given in Section IX—Substructures.

*"Beams with Intermediate Expansion Hinges in Rigid-Frame Bridges" by D. H. Pletta and Leonard C. Hollister, *Journal American Concrete Institute*, January, 1939, page 149; and *Beam Factors and Moment Coefficients for Members with Intermediate Expansion Hinges* published by Portland Cement Association.

Soucook River Bridge near Loudon, N. H., designed by New Hampshire State Highway Department; John W. Childs, bridge engineer. The deck, 28 ft. 10 in. wide, is carried on three girders 9 ft. 6 in. apart.

An example of solid piers built integral with the deck is the Salinas River Bridge at Soledad, Calif., consisting of two 89-ft. and thirteen 104-ft. spans. Designed by California State Division of Highways; F. W. Panhorst, bridge engineer.

Pile Bents

Reinforced concrete pile bents may be used wherever they can be driven, except where pier reactions are too great for the maximum size and minimum spacing of such piles generally considered good practice; or where ice jams or drift at flood stage require closed piers.

Reinforced concrete piles serve well for open abutments. When filled embankments are over 10 ft. high, roadway approach slabs should be used with pile bent open abutments.

Open Frame Bents

Open frame bents may be used where concrete piles cannot be driven or where reactions are too great for pile bents, or where location of proposed bridge is inaccessible to the pile driving equipment required for concrete piles. Open frame bents, likewise, are well suited to serve as abutments. As in the case of pile bents, if the embankment is over 10 ft. high roadway approach slabs should be used.

Solid Piers

Solid piers are required when mass is needed to resist high wind forces, ice jams, exceptionally heavy drift, or for protection against collision of traffic under bridge.

Closed Abutments

Closed abutments may be required where crossroads or streets, railroad tracks or other construction prevent the necessary lengthening of the bridge for open abutments. Their use at other places will, in general, be less economical than the open types.

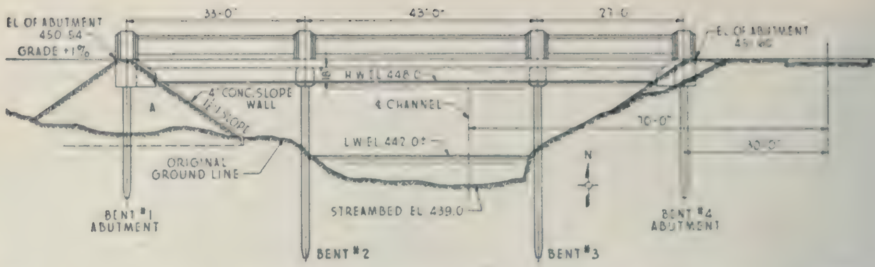

Fig. 1. Three-span unsymmetrical continuous slab bridge—Layout Problem No. 1.

LAYOUT PROBLEM No. 1

Problem: To lay out a highway bridge over a small stream for which the site conditions are:

1. Waterway area required 630 sq.ft.
2. Depth of flow at high-water 9 ft.
3. Cross-section profile as shown in Fig. 1.
4. Not subject to violent floods; only small amount of drift.
5. Subsoil will permit driving piles.
6. Flood plain extends 120 ft. west of centerline of channel.
7. Fill at point 60 ft. west of centerline of channel 7 ft. above original ground line and 3 ft. at point 40 ft. east of centerline.
8. Crossroad 70 ft. east of centerline of channel.
9. Eighteen inches estimated as required clearance above high-water.

The conditions of the problem are met satisfactorily by a three-span continuous slab structure with uniform depth of slab (spans 33 ft.-43 ft. -27 ft.) having a pile bent substructure with a 4-in. concrete slope wall at abutment Bent No. 1 as shown in Fig. 1 because:

1. Layout provides 650-sq.ft. waterway.
2. Subsoil conditions combined with absence of floods and small amount of drift make concrete pile bents the logical choice for the substructure.
3. Rough comparative estimates show a 25 per cent reduction in cost of proposed layout over that of a single span of shorter length with closed abutments on foundation piles—the next logical choice.
4. Location of the crossroad fixes location of Bent No. 4 at point shown to allow room for turnout.
5. Bents No. 2 and No. 3 were shifted 3 ft. east of the positions that would give a symmetrical layout in order to place Bent No. 3 at a more satisfactory point with reference to the bank of the stream.
6. Flood plain is restricted to a minimum consistent with economy and requires no channel excavation—any appreciable channel excavation is likely to be of uncertain value.
7. Four-inch concrete slope wall is provided at open end Bent No. 1 to prevent erosion of fill which might otherwise occur because of

the restriction of the flow at flood stage. Backfill *A* is to be placed and firmly compacted and slope wall placed before deck is constructed.

8. The deck slab is made of constant depth since for such small span lengths no saving can be made by using a variable depth slab. An improvement in appearance, though, can be had at very little increase in cost by using a parabolic haunched slab.

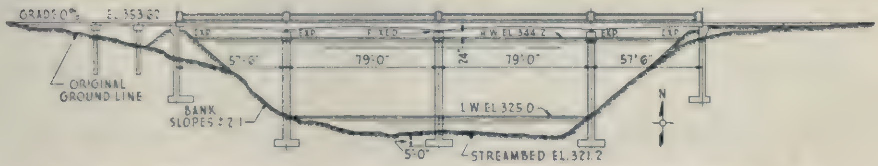

Fig. 2. Four-span symmetrical T-girder bridge—Layout Problem No. 2.

LAYOUT PROBLEM No. 2

Conditions at site of proposed bridge are:

1. Waterway required 4,200 sq.ft.
2. Depth of flow at high water ± 23 ft.
3. Flood plain is confined within banks of stream.
4. Soil, tough yellow clay, mixed with some sand and gravel.
5. Banks of stream heavily wooded upstream.
6. Ice gorges at site not anticipated.
7. Elevation of bed of stream known to fluctuate very little.
8. Channel straight at this point and does not shift appreciably.
9. Stream carries some drift at high-water.

Proposed layout, Fig. 2, was selected for the following reasons:

1. Over-all length with end slopes slightly greater than 1.5:1 conforms closely to stream cross section, and furnishes the required waterway.
2. Foundation soil considered capable of sustaining 4 tons per sq.ft. without objectionable settlement, therefore permits the use of open bent piers on spread footings.
3. Chosen layout places two bents at low waterline, leaving only one pier in main body of stream, which while not desirable, is not considered serious enough to justify the extra cost of spanning stream with one span.
4. Span ratios 1:1.37:1.37:1 provide maximum economy in deck and symmetrical reinforcing layout.
5. Since bank erosion is not expected to become serious, no slope walls are specified, and elevations of abutment footings may be placed appreciably higher than those of the piers.

Layout Problems Nos. 1 and 2 will later be used as examples to demonstrate the design procedure.

Twelve-span continuous girder bridge over Kankakee River, Will County, Ill., consists of three groups of four 83-ft. spans. Designed by Illinois Division of Highways; George F. Burch, engineer of bridges.

Section III—Loading

The A.A.S.H.O.* provides that a truck-train loading, Fig. 3, or an equivalent lane loading consisting of a uniform load and a single concentrated load be used for the design of continuous bridges, whichever loading gives the greater stresses. This may necessitate the determination of moments and shears at some sections for both loadings, since for all combinations of spans the lane loading is not truly equivalent to the truck-train loading. The procedure for analysis is the same, regardless of the type of loading, but requires somewhat less time with the equivalent loading, since only one moving concentrated load is involved. However, for the purpose of illustrating design procedure, in the examples which follow the truck-train loading will be used.

Fortunately, moments and shears of continuous units for truck-train loading can be found very readily by the use of influence lines. Consequently, the method of influence lines is used in this booklet.

Distribution of Wheel Loads

A strictly accurate method of distributing wheel loads across bridge decks is not yet available. For all practical purposes, however, satisfactory results are obtained if wheel loads on solid slabs are distributed in accordance with the formula $E = 0.135S + 3.2$, in which E = the transverse width in feet (maximum 6 ft.) over which one wheel is to be distributed and S = length of the loaded span in feet. This formula is based upon a 10-ft. traffic lane** and is a modification of a formula which appeared in a bulletin***

*See Standard Specifications for Highway Bridges, adopted by The American Association of State Highway Officials, 1949, section 3.2.8 and subsequent sections.

**See *Public Roads*, September, 1937, page 129.

***In the bulletin, *Distribution of Wheel Loads and Design of Reinforced Concrete Bridge Floor Slabs*, published July, 1934 by the Bureau of Public Roads (see also *Public Roads*, October, 1937, page 149), the equivalent effective width for slabs on three or more supports and continuous over two or more panels is given as $E = 0.120S + 3.1$ for 9-ft. traffic lanes. The formula given here for 10-ft. traffic lanes is that used by the Illinois Division of Highways.

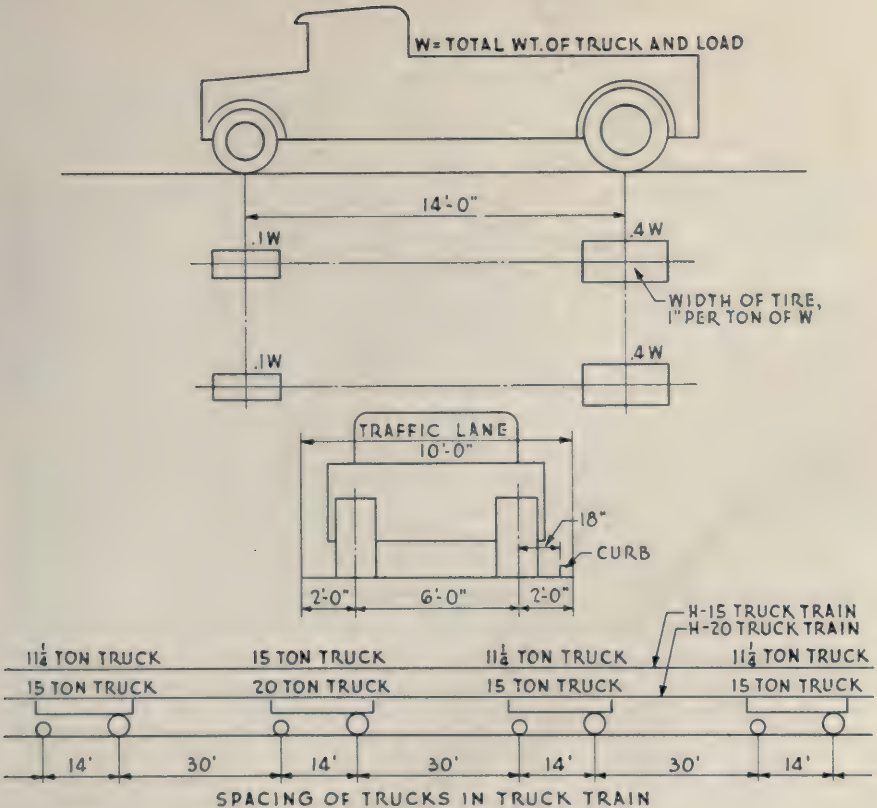

Fig. 3. A.A.S.H.O. truck-train loading.

issued by the Bureau of Public Roads on wheel load distribution for a 9-ft. lane.

In the design of girder bridges, considering a 10-ft. traffic lane, the fraction of a wheel load carried by each girder is $\frac{S_1}{5}$, in which S_1 = center to center spacing of girders in feet.

As far as is ascertainable, these formulas have given satisfactory results.

Long spans possible with continuous girder bridges mean less interference with stream flow. Bear Creek Bridge near Marshall, Ark., designed by Arkansas State Highway Commission; N. B. Garver, principal highway engineer.

Section VI—Design Method

Moment Distribution

As mentioned in the Foreword, the method of analysis presented here is based upon the concept of moment distribution developed by Prof. Hardy Cross*. Briefly stated, the procedure for determining final moments in a continuous structure by the moment distribution method is as follows:

The initial step is to consider all joints fixed or locked against rotation, thereby confining the effect of the load in any span to the supporting beam, and determine the fixed end moments.

The next step is to release or unlock one of the joints and distribute the unbalanced moment at that joint to all of the members integrally connected at the joint in accordance with their relative stiffnesses. When this operation is completed, the joint at which moments have just been distributed is relocked in its rotated position and procedure repeated at succeeding joints.

With the release of a joint and the distribution of the unbalanced moment at that joint to the adjacent members, a moment is induced or carried over to the far end of those members. The moments thus carried over are comparable to a new set of fixed end moments and again create a condition of unbalanced moments at the joints which must be distributed.

By repeating the foregoing process, the carried-over moments become less and less, the unbalanced moments smaller and smaller, and the algebraic sums of the original fixed end moments plus the distributed moments and the carried-over moments approach the exact final moments at the joints. The more cycles through which the procedure is carried, the greater will be the accuracy of the final result.

The method is neither laborious nor difficult when only uniform loads or fixed concentrated loads are involved, but becomes so when moving loads must be considered and the position for maximum moments must be determined. Therefore, in order to determine final moments at supports of continuous bridges without the necessity for carrying out the usual procedure step by step, formulas giving the final distributed moments for loads in any span have been derived for two, three and four-span units** in which the deck is freely supported at the abutments and either freely supported at the intermediate piers or integral with them. These formulas, given in Table I for deck loads and Table II for changes in deck length, have been derived as the summation of an infinite series of distributions and consequently give exact results assuming that sidesway is prevented***. The accuracy and use of the formulas will be demonstrated later. Also, a method

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*"Analysis of Continuous Frames by Distributing Fixed End Moments" by Hardy Cross, *Transactions A.S.C.E.*, Vol. 96, pages 1-156, and *Moment Distribution Applied to Continuous Concrete Structures*, published by Portland Cement Association.

**The complete derivation of these formulas will be furnished upon request to the Portland Cement Association.

***It is customary to assume that sidesway is prevented because when a single lane only is loaded the remaining unloaded lanes as well as the abutments, the highway slab and the approach fills all combine to restrain the structure against sidesway.

for obtaining moments in four-span unsymmetrical bridges and structures of five or more spans, whether symmetrical or unsymmetrical, will be discussed in a subsequent section.

Fixed End Moments and Beam Constants

The formulas for final moments involve the terms encountered in the customary moment distribution procedure, namely:

M^F = fixed end moments (negative for deck loads and deck lengthening and positive for deck shortening)

C = carry-over factors (always negative)

D = distribution factors (always positive)

Fixed end moments and the beam constants for carry-over and stiffness (distribution factors depend upon the latter) are dependent upon the manner in which the moment of inertia of the member varies between supports. Curves for these factors (Figs. 5 to 17) have been prepared for symmetrical and unsymmetrical members with soffit curves made up of arcs of two parabolas, but having the same apex at centerline of span; one arc passes through support A at $r_A h_c$ below the apex and the other passes through support B at $r_B h_c$ below the apex. As shown in Fig. 4,

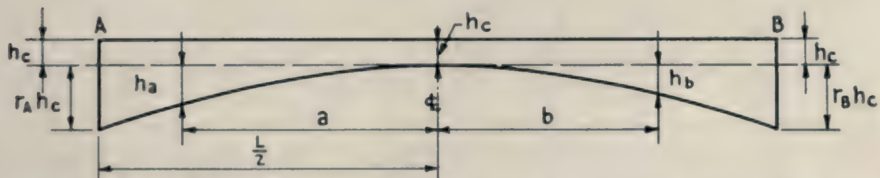

Fig. 4. Soffit curves are arcs of two parabolas having same apex at centerline of span

h_a and h_b = offsets to the parabolic soffit at a distance a and b from centerline of span

and

$$h_a = \frac{a^2 \times r_A h_c}{\left(\frac{L}{2}\right)^2} \qquad h_b = \frac{b^2 \times r_B h_c}{\left(\frac{L}{2}\right)^2}$$

From the above equations the total depth of deck at any point in a span may be found by adding the values of h_a or h_b to h_c which is a constant.

Tables and curves for beam constants for symmetrical members with straight and sharply curved soffits and for unsymmetrical members with straight haunches at one end are published elsewhere*. For beams of irregular or discontinuous variation in moment of inertia, the constants may be obtained by column analogy** or other methods***.

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*_Structural Frameworks_, by T. H. Hickerson, University of North Carolina Press; _Analysis of Continuous Frames by the Method of Restraining Stiffness_ by E. B. Russell, published by Ellison and Russell, San Francisco, Calif., 1934; and _Handbook of Frame Constants_, published by Portland Cement Association.

**_Column Analogy_ by Hardy Cross, Univ. of Illinois Experiment Station, Bulletin 215.

***_Transactions A.S.C.E._, Vol. 96, page 101, discussion by George Large; and Ohio State University Engineering Experiment Station, Bulletin 66.

The charts for beam constants and fixed end moments are for solid slab decks and have been prepared by using parameters r_A and r_B which are the ratios of the increase in depth of the deck at the supports to the depth at the centerline. Taking each span separately, the parameters may be expressed algebraically:

$$r_A = \frac{h_A - h_c}{h_c} \qquad r_B = \frac{h_B - h_c}{h_c} \tag{8}$$

in which h_A = depth at left support of span AB
h_B = depth at right support of span AB
h_c = depth at centerline of span AB

Although the charts are for solid slab decks, they may be used also for T-girders with parabolic soffits since the variation in moment of inertia of the members is such that they can be reduced to an equivalent slab with a loss in accuracy generally of less than 1 per cent. While the variation in moment of inertia for T-girders and other types may be practically the same as for solid slabs with parabolic soffits, the r values cannot be used directly to find depths; the r values are, in such cases, merely a measure of the magnitude of the variation of the moment of inertia, in other words parametric constants. Values of r_A and r_B for an equivalent slab may be obtained by the following formulas:

$$r_A = \sqrt[3]{\frac{I_A}{I_c}} - 1 \qquad r_B = \sqrt[3]{\frac{I_B}{I_c}} - 1 \tag{9}$$

in which

I_A and I_B = moment of inertia of T-girder cross section at ends A and B respectively

I_c = moment of inertia of T-girder cross section at centerline of span

Embankment lines are not interrupted where open abutments are used, which improves appearance of the roadway; visibility is better, and cost is reduced by eliminating heavy retaining walls. Highway grade separation southeast of Aurora, Ill., designed by Illinois Division of Highways; Alfred Benesch, engineer of grade separations.

Values of I_A, I_B and I_c are for the gross section of the beam, neglecting the reinforcement*. The reinforcement will increase the moment of inertia, but since only relative values are required it has no appreciable effect.

Distribution Factors

Each distribution factor, D_{AB}, D_{BA}, D_{BC}, ... , is obtained as a ratio of the stiffness at the end of member to the sum of the stiffnesses of all the members intersecting at the joint, including the support if the deck is integrally connected to it, or

$$D = \frac{K}{\Sigma K} \equiv \frac{\frac{kI_cE}{L}}{\Sigma\frac{kI_cE}{L}} \tag{10}$$

in which

k = stiffness coefficients of joint end of members taken from accompanying curves, Fig. 6

L = span length of members

I_c = moment of inertia at the centerline of the members involved, unless otherwise specified**

E = modulus of elasticity of concrete which is usually the same throughout the structure and can therefore be cancelled.

K = stiffness of a member which is defined as the moment required to rotate the simply supported end of a member through a unit angle when the other end is fixed

The stiffness coefficients taken directly from the curves, Fig. 6, are for members continuous at both ends and apply to the interior spans of a bridge. Since, however, the exterior spans are usually discontinuous, in other words are not built integral with the abutments, it is necessary to correct the stiffness values taken from the curves in order to make them applicable to such members. It can be shown *** that the stiffness coefficient at the continuous end of a beam AB which is discontinuous at A is

$$k = (1 - C_{AB}C_{BA})k_{BA} \tag{11}$$

in which

k_{BA} = stiffness coefficient taken from curve

C_{AB} and C_{BA} = carry-over factors at A and B of member AB

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*University of Illinois, Engineering Experiment Station, *Bulletin 308*, page 74, paragraph 18, (b).

**When k values are taken from sources other than this booklet it is necessary to use the moment of inertia of the section of the member which is the section of reference for the particular chart or table used.

***See *One-Story Concrete Frames Analyzed by Moment Distribution*, page 4, published by Portland Cement Association.

TABLE I - FINAL MOMENTS AT SUPPORTS DUE TO DECK LOADS*

TWO SPANS - SYMMETRICAL OR UNSYMMETRICAL

SECTION	FIRST SPAN LOADED	SECOND SPAN LOADED
M_{AB}	0	0
M_{BA}	$(1-D_{BA})\,M_1$	$D_{BA}\,M_2$
M_{BC}	$D_{BC}\,M_1$	$(1-D_{BC})\,M_2$
M_{CB}	0	0

THREE SPANS - SYMMETRICAL OR UNSYMMETRICAL

SECTION	FIRST SPAN LOADED	SECOND SPAN LOADED	THIRD SPAN LOADED
M_{AB}	0	0	0
M_{BA}	$\frac{1-D_{BA}-U}{1-U}M_1$	$\frac{D_{BA}M^F_{BC}-WM^F_{CB}}{1-U}$	$\frac{W}{1-U}M_3$
M_{BC}	$\frac{D_{BC}-U}{1-U}M_1$	$\frac{(1-D_{BC})M^F_{BC}-C_{CB}D_{CB}(1-D_{BC})M^F_{CB}}{1-U}$	$\frac{C_{CB}D_{CB}(1-D_{BC})}{1-U}M_3$
M_{CB}	$\frac{C_{BC}D_{BC}(1-D_{CB})}{1-U}M_1$	$\frac{(1-D_{CB})M^F_{CB}-C_{BC}D_{BC}(1-D_{CB})M^F_{BC}}{1-U}$	$\frac{D_{CB}-U}{1-U}M_3$
M_{CD}	$\frac{V}{1-U}M_1$	$\frac{D_{CD}M^F_{CB}-VM^F_{BC}}{1-U}$	$\frac{1-D_{CD}-U}{1-U}M_3$
M_{DC}	0	0	0

FOUR SPANS - SYMMETRICAL ABOUT CENTER PIER**

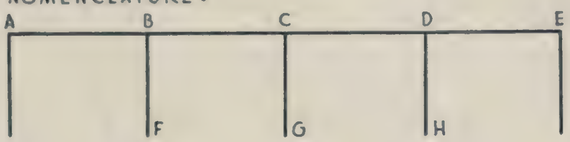
SECTION	FIRST SPAN LOADED	SECOND SPAN LOADED	THIRD SPAN LOADED	FOURTH SPAN LOADED
M_{AB}	0	0	0	0
M_{BA}	$\frac{(1-D_{BA})-(2-D_{BA})U}{1-2U}M_1$	$\frac{D_{BA}(1-U)M^F_{BC}-WM^F_{CB}}{1-2U}$	$\frac{-UD_{DE}M^F_{DC}+WM^F_{CD}}{1-2U}$	$\frac{UD_{DE}}{1-2U}M_4$
M_{BC}	$\frac{D_{BC}(1-U)-U}{1-2U}M_1$	$\frac{(1-D_{BC})(1-U)M^F_{BC}-C_{CB}D_{CB}(1-D_{BC})M^F_{CB}}{1-2U}$	$\frac{-U(1-D_{DC})M^F_{DC}+C_{CD}D_{CD}(1-D_{DC})M^F_{CD}}{1-2U}$	$\frac{U(1-D_{DC})}{1-2U}M_4$

M_{CB}	$\frac{C_{BC}D_{BC}(1-D_{CB}-U)}{1-2U}M_1$	$\frac{-C_{BC}D_{BC}(1-D_{CB}-U)M^F_{BC}+(1-D_{CB}-U)M^F_{CB}}{1-2U}$	$\frac{-C_{DC}D_{DC}(D_{CD}-U)M^F_{DC}+(D_{CD}-U)M^F_{CD}}{1-2U}$	$\frac{C_{DC}D_{DC}(D_{CD}-U)}{1-2U}M_4$
M_{CD}	$\frac{C_{BC}D_{BC}(D_{CB}-U)}{1-2U}M_1$	$\frac{-C_{BC}D_{BC}(D_{CB}-U)M^F_{BC}+(D_{CB}-U)M^F_{CB}}{1-2U}$	$\frac{-C_{DC}D_{DC}(1-D_{CD}-U)M^F_{DC}+(1-D_{CD}-U)M^F_{CD}}{1-2U}$	$\frac{C_{DC}D_{DC}(1-D_{CD}-U)}{1-2U}M_4$
M_{DC}	$\frac{U(1-D_{BC})}{1-2U}M_1$	$\frac{-U(1-D_{BC})M^F_{BC}+C_{CB}D_{CB}(1-D_{BC})M^F_{CB}}{1-2U}$	$\frac{(1-D_{DC})(1-U)M^F_{DC}-C_{CD}D_{CD}(1-D_{DC})M^F_{CD}}{1-2U}$	$\frac{D_{DC}(1-U)-U}{1-2U}M_4$
M_{DE}	$\frac{UD_{BA}}{1-2U}M_1$	$\frac{-UD_{BA}M^F_{BC}+WM^F_{CB}}{1-2U}$	$\frac{D_{DE}(1-U)M^F_{DC}-WM^F_{CD}}{1-2U}$	$\frac{(1-D_{DE})-(2-D_{DE})U}{1-2U}M_4$
M_{ED}	0	0	0	0

*DECK FREELY SUPPORTED AT ABUTMENTS AND EITHER INTEGRAL WITH OR FREELY SUPPORTED AT INTERMEDIATE PIERS. WHEN DECK AND PIERS ARE INTEGRAL, PIER MOMENTS MAY BE OBTAINED BY SUBTRACTING THE MOMENT IN THE DECK AT ONE SIDE OF THE PIER FROM THE MOMENT IN THE DECK AT THE OTHER SIDE. THE PIER MOMENT WILL ACT IN THE SAME DIRECTION AS THE SMALLER DECK MOMENT.

**WHEN PIERS AND DECK ARE INTEGRAL, PIER HEIGHTS AND SECTIONS AS WELL AS DECK SPANS AND SECTIONS ARE SYMMETRICAL.

NOMENCLATURE:



M_{AB}, M_{BA}, M_{BC}, ETC., = FINAL MOMENTS

M^F_{AB}, M^F_{BA}, M^F_{BC}, ETC., = FIXED END MOMENTS

C_{AB}, C_{BA}, C_{BC}, ETC., = CARRY OVER FACTORS

D_{AB}, D_{BA}, D_{BC}, ETC., = DISTRIBUTION FACTORS

IN EACH OF THE ABOVE SYMBOLS THE FIRST SUBSCRIPT LETTER INDICATES THE POINT UNDER CONSIDERATION IN THE MEMBER DESIGNATED BY THE TWO SUBSCRIPTS.

EQUIVALENTS:

(1) $M_1 = M^F_{BA} - C_{AB} M^F_{AB}$

(2) $M_2 = M^F_{BC} - C_{CB} M^F_{CB}$

(3) $M_3 = M^F_{CD} - C_{DC} M^F_{DC}$

(4) $M_4 = M^F_{DE} - C_{ED} M^F_{ED}$

(5) $U = C_{BC} C_{CB} D_{BC} D_{CB} = C_{DC} C_{CD} D_{DC} D_{CD}$
(BY SYMMETRY IN FOUR SPAN STRUCTURES)

(6) $V = C_{BC} D_{BC} D_{CD}$

(7) $W = C_{CB} D_{CB} D_{BA} = C_{CD} D_{CD} D_{DE}$
(BY SYMMETRY IN FOUR SPAN STRUCTURES)

TABLE II - FINAL MOMENTS AT SUPPORTS DUE TO CHANGES IN DECK LENGTH WHEN DECK AND INTERIOR PIERS ARE INTEGRAL

THREE SPANS – SYMMETRICAL OR UNSYMMETRICAL				
SECTION	DISPLACEMENT OF LEFT PIER		DISPLACEMENT OF RIGHT PIER	
M_{AB}	0		0	
M_{BA}	$-\frac{D_{BA}}{1-U} M^F_{BF}$		$\frac{C_{CB} D_{CB} D_{BA}}{1-U} M^F_{CG}$	
M_{BF}	$\frac{1-D_{BF}-U}{1-U} M^F_{BF}$		$\frac{C_{CB} D_{CB} D_{BF}}{1-U} M^F_{CG}$	
M_{BC}	$\frac{D_{BC}-U}{1-U} M^F_{BF}$		$\frac{C_{CB} D_{CB} (1-D_{BC})}{1-U} M^F_{CG}$	
M_{FB}	0**	$M^F_{FB}+(M^F_{BF}-M_{BF})C_{BF}$***	0**	$C_{BF} M_{BF}$***
M_{GC}	0**	$C_{CG} M_{CG}$***	0**	$M^F_{GC}+(M^F_{CG}-M_{CG})C_{CG}$***
M_{CB}	$\frac{C_{BC} D_{BC} (1-D_{CB})}{1-U} M^F_{BF}$		$\frac{D_{CB}-U}{1-U} M^F_{CG}$	
M_{CG}	$\frac{C_{BC} D_{BC} D_{CG}}{1-U} M^F_{BF}$		$\frac{1-D_{CG}-U}{1-U} M^F_{CG}$	
M_{CD}	$\frac{C_{BC} D_{BC} D_{CD}}{1-U} M^F_{BF}$		$-\frac{D_{CD}}{1-U} M^F_{CG}$	
M_{DC}	0		0	

FOUR SPANS – SYMMETRICAL ABOUT CENTER PIER*		
SECTION	EQUAL DISPLACEMENT OF LEFT AND RIGHT PIERS	
M_{AB}	0	
M_{BA}	$-D_{BA} M^F_{BF}$ OR $-D_{BA} M^F_{DH}$	
M_{BF}	$(1-D_{BF}) M^F_{BF}$ OR $(1-D_{BF}) M^F_{DH}$	
M_{BC}	$D_{BC} M^F_{BF}$ OR $D_{BC} M^F_{DH}$	
M_{CB}	$C_{BC} D_{BC} M^F_{BF}$ OR $C_{BC} D_{BC} M^F_{DH}$	
M_{CG}	0	
M_{FB}	0**	$M^F_{FB}+(M^F_{BF}-M_{BF})C_{BF}$***
M_{GC}	0**	0***
M_{HD}	0**	$M^F_{HD}+(M^F_{DH}-M_{DH})C_{DH}$***
M_{CD}	$C_{DC} D_{DC} M^F_{BF}$ OR $C_{DC} D_{DC} M^F_{DH}$	
M_{DC}	$D_{DC} M^F_{BF}$ OR $D_{DC} M^F_{DH}$	
M_{DH}	$(1-D_{DH}) M^F_{BF}$ OR $(1-D_{DH}) M^F_{DH}$	
M_{DE}	$-D_{DE} M^F_{BF}$ OR $-D_{DE} M^F_{DH}$	
M_{ED}	0	

*PIER HEIGHTS AND SECTIONS AS WELL AS DECK SPANS AND SECTIONS ARE SYMMETRICAL ABOUT CENTER PIER

** PIERS HINGED AT BOTTOM

*** PIERS FIXED AT BOTTOM

NOMENCLATURE AND EQUIVALENTS SAME AS TABLE I

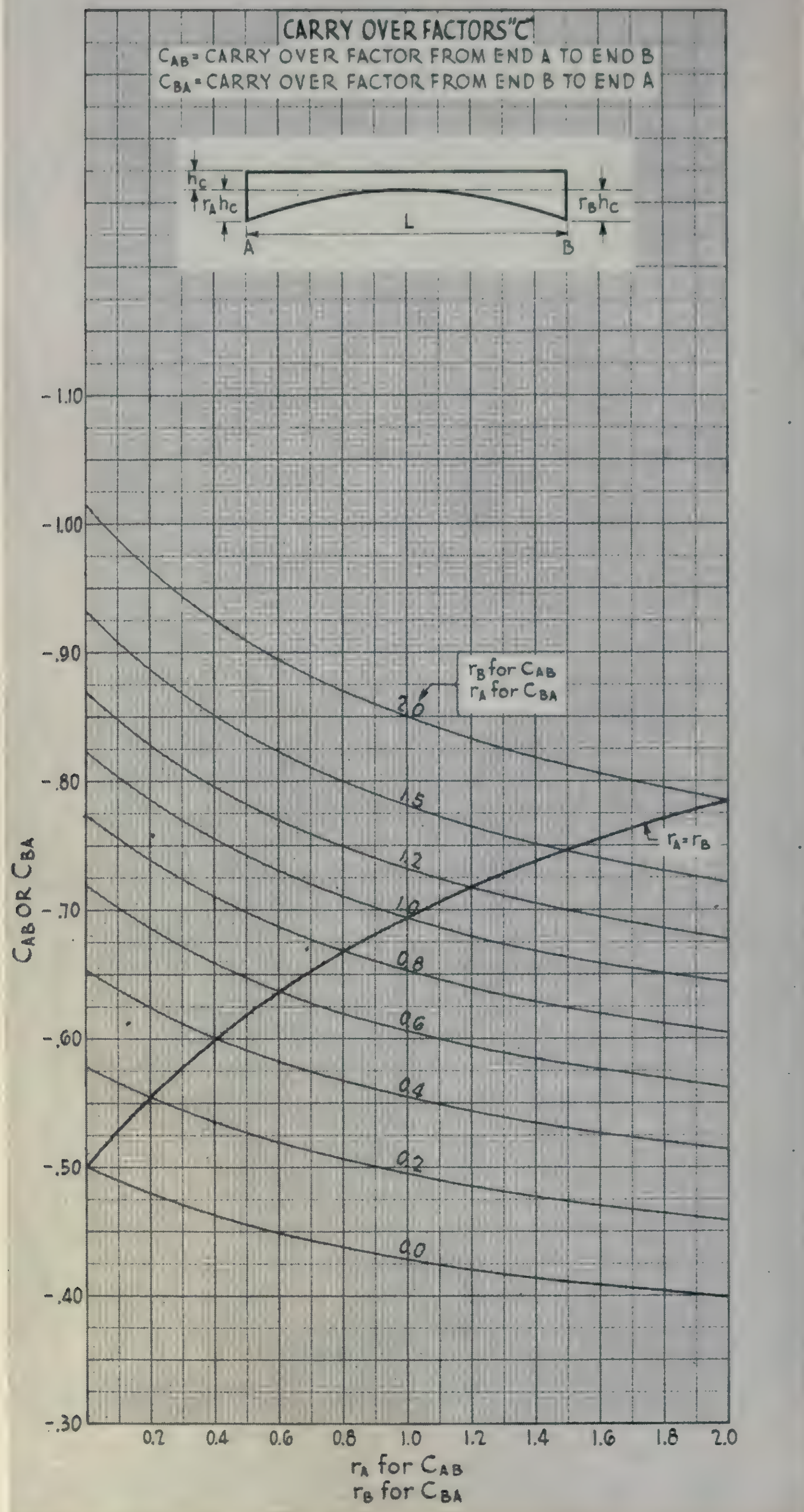

CARRY OVER FACTORS "C"
C_{AB} = CARRY OVER FACTOR FROM END A TO END B
C_{BA} = CARRY OVER FACTOR FROM END B TO END A
h_c
$r_A h_c$
$r_B h_c$
L
A
B
r_B for C_{AB}
r_A for C_{BA}
$r_A = r_B$
2.0
1.5
1.2
1.0
0.8
0.6
0.4
0.2
0.0
C_{AB} OR C_{BA}
-1.10
-1.00
-.90
-.80
-.70
-.60
-.50
-.40
-.30
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
r_A for C_{AB}
r_B for C_{BA}

Fig. 5

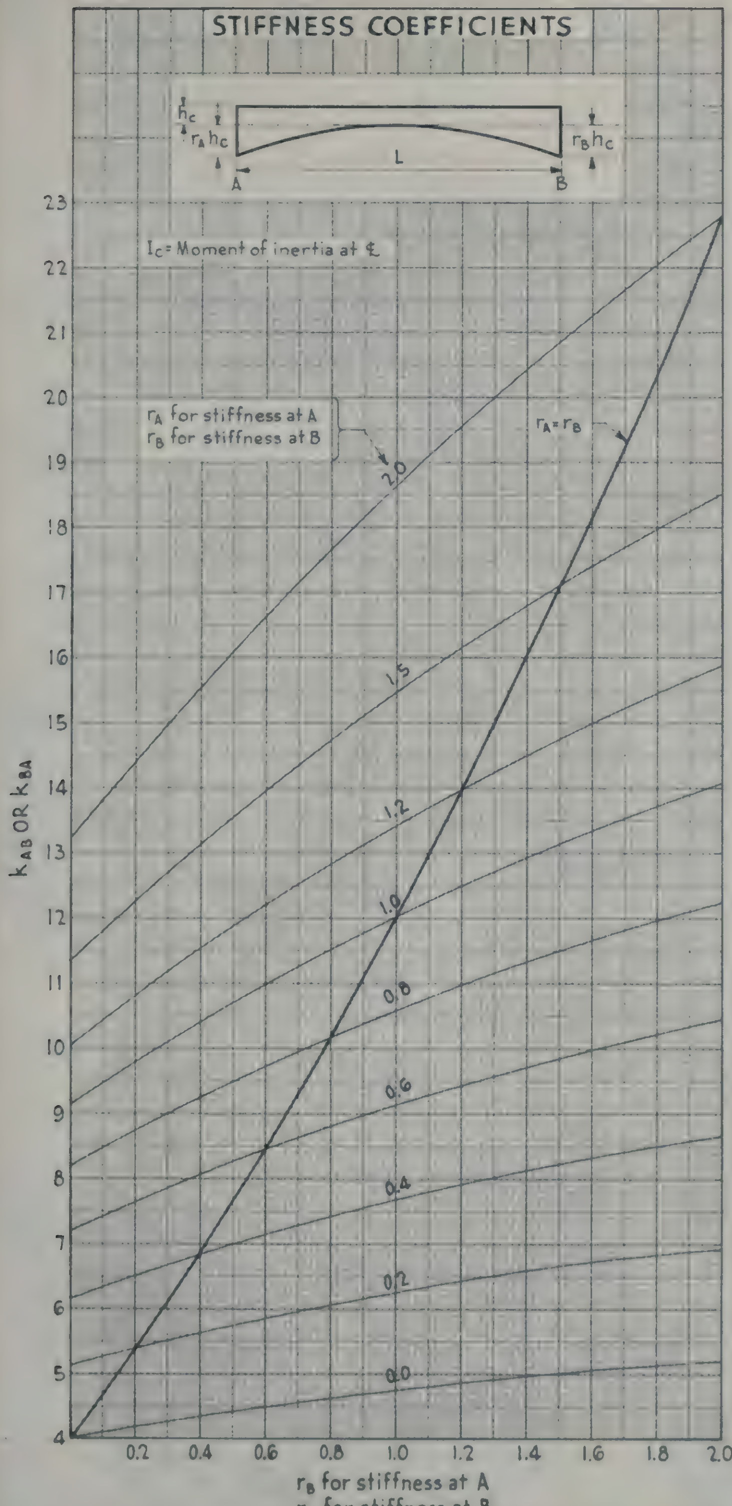

STIFFNESS COEFFICIENTS
h_c
$r_A h_c$
$r_B h_c$
L
A
B
I_c = Moment of inertia at ℄
r_A for stiffness at A
r_B for stiffness at B
$r_A = r_B$
2.0
1.5
1.2
1.0
0.8
0.6
0.4
0.2
0.0
k_{AB} OR k_{BA}
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
r_B for stiffness at A
r_A for stiffness at B

Fig. 6

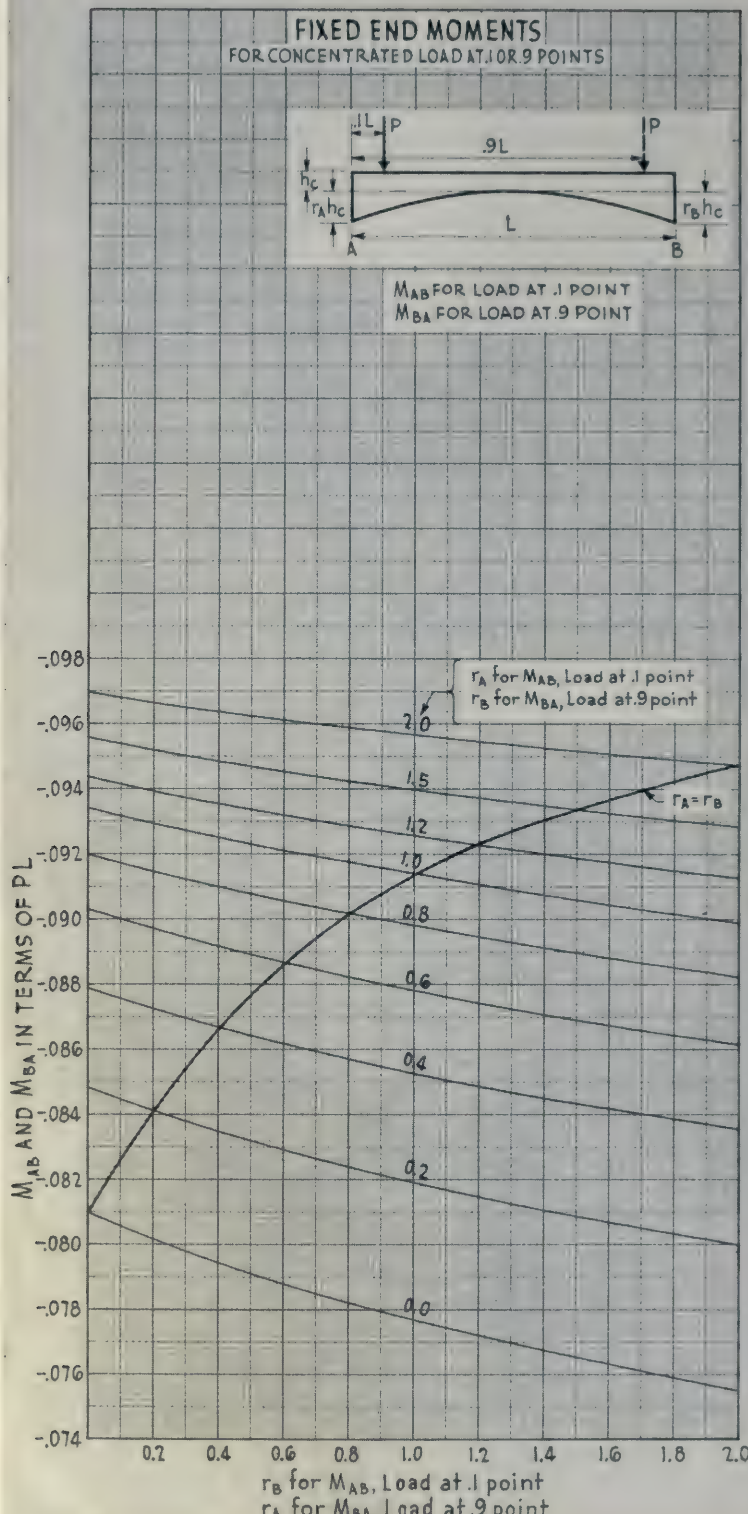

FIXED END MOMENTS
FOR CONCENTRATED LOAD AT .1 OR .9 POINTS
.1L
P
P
.9L
h_c
$r_A h_c$
$r_B h_c$
L
A
B
M_{AB} FOR LOAD AT .1 POINT
M_{BA} FOR LOAD AT .9 POINT
r_A for M_{AB}, Load at .1 point
r_B for M_{BA}, Load at .9 point
2.0
1.5
1.2
1.0
0.8
0.6
0.4
0.2
0.0
$r_A = r_B$
M_{AB} AND M_{BA} IN TERMS OF PL
-.098
-.096
-.094
-.092
-.090
-.088
-.086
-.084
-.082
-.080
-.078
-.076
-.074
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
r_B for M_{AB}, Load at .1 point
r_A for M_{BA}, Load at .9 point

Fig. 7

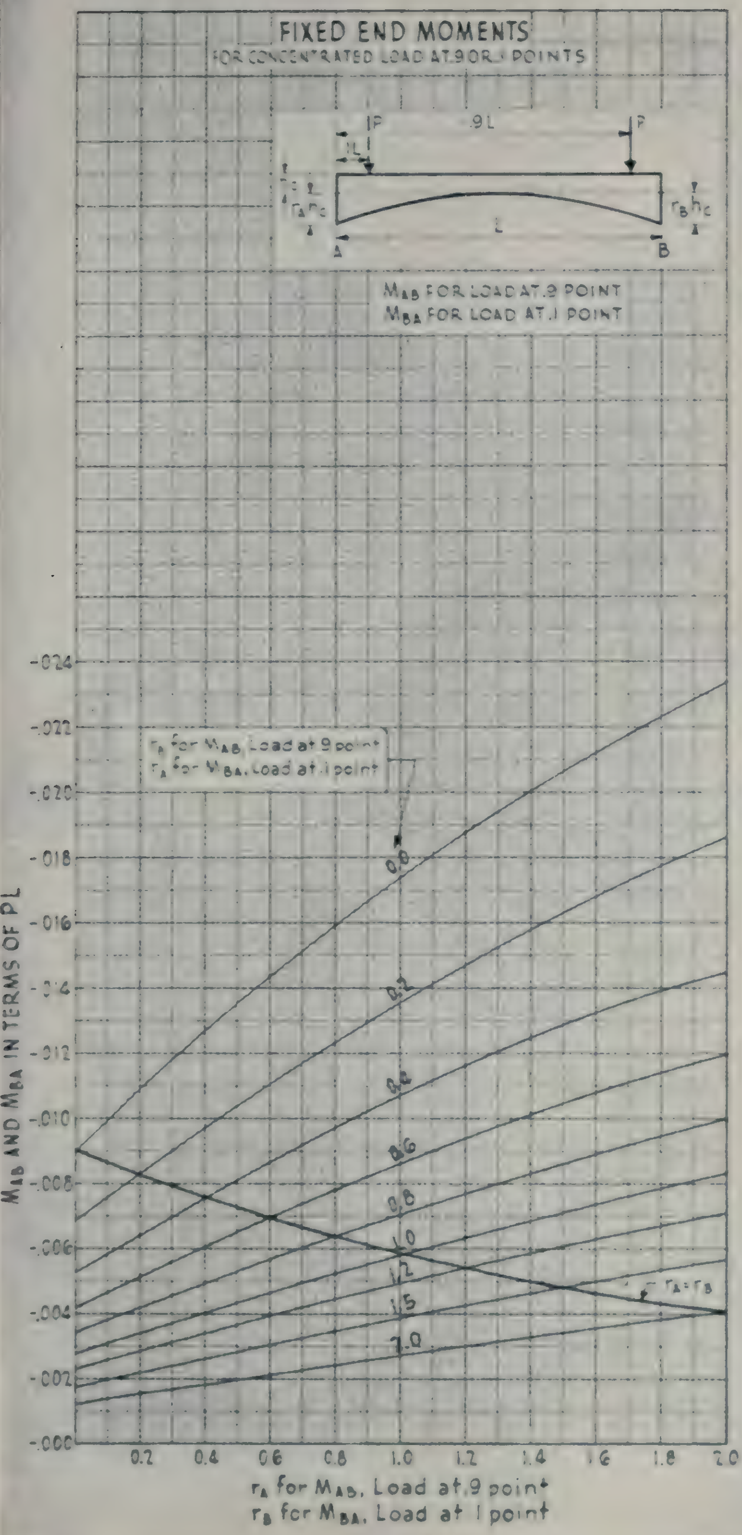

FIXED END MOMENTS
FOR CONCENTRATED LOAD AT .9 OR .1 POINTS
P
.9L
P
.1L
L
A
B
$r_A h_c$
$r_B h_c$
M_{AB} FOR LOAD AT .9 POINT
M_{BA} FOR LOAD AT .1 POINT
r_B for M_{AB}, Load at .9 point
r_A for M_{BA}, Load at .1 point
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.5
2.0
$r_A = r_B$
M_{AB} AND M_{BA} IN TERMS OF PL
-.024
-.022
-.020
-.018
-.016
-.014
-.012
-.010
-.008
-.006
-.004
-.002
-.000
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
r_A for M_{AB}, Load at .9 point
r_B for M_{BA}, Load at .1 point

Fig. 8

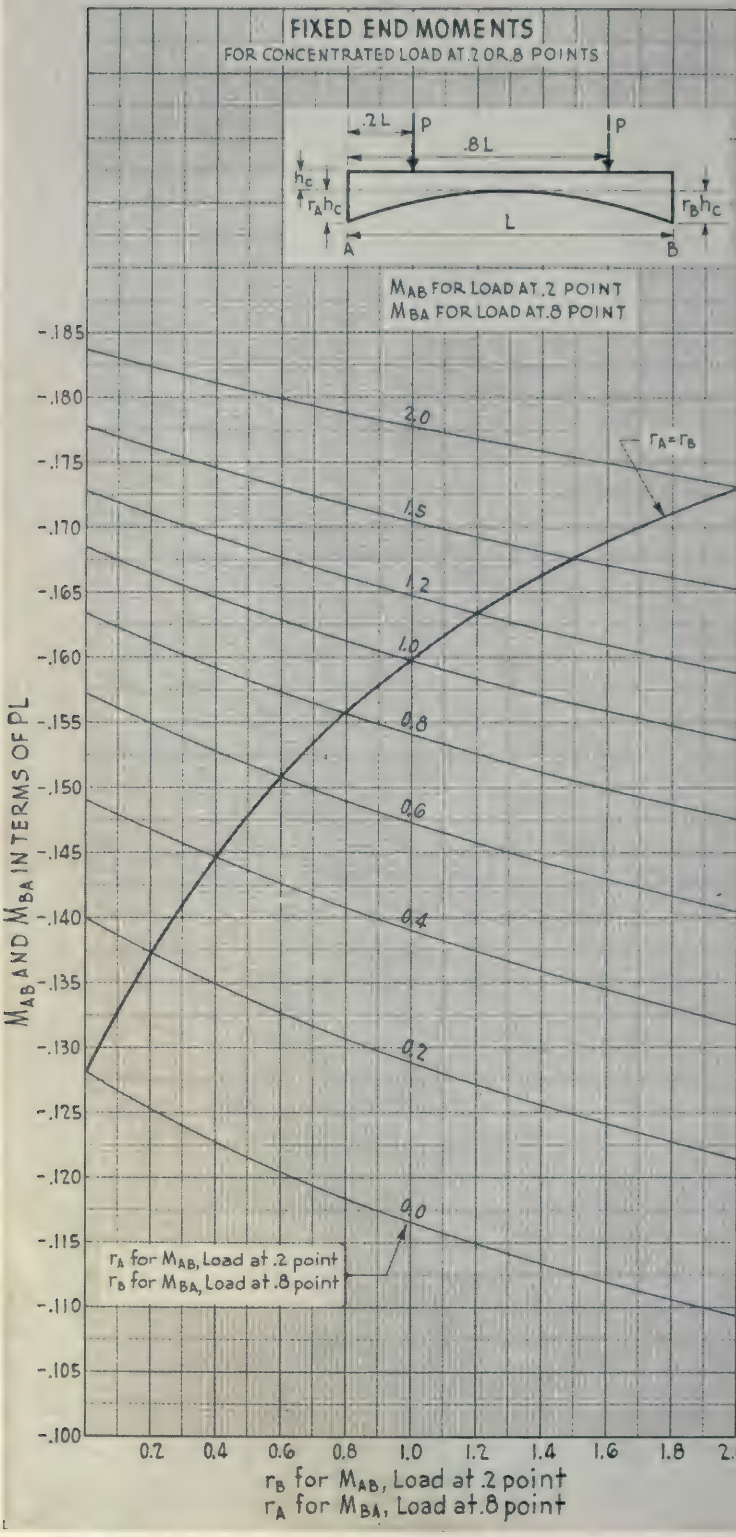

FIXED END MOMENTS
FOR CONCENTRATED LOAD AT .2 OR .8 POINTS
.2L
P
P
.8L
h_c
$r_A h_c$
$r_B h_c$
L
A
B
M_{AB} FOR LOAD AT .2 POINT
M_{BA} FOR LOAD AT .8 POINT
$r_A = r_B$
2.0
1.5
1.2
1.0
0.8
0.6
0.4
0.2
0.0
r_A for M_{AB}, Load at .2 point
r_B for M_{BA}, Load at .8 point
M_{AB} AND M_{BA} IN TERMS OF PL
-.185
-.180
-.175
-.170
-.165
-.160
-.155
-.150
-.145
-.140
-.135
-.130
-.125
-.120
-.115
-.110
-.105
-.100
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
r_B for M_{AB}, Load at .2 point
r_A for M_{BA}, Load at .8 point

Fig. 9

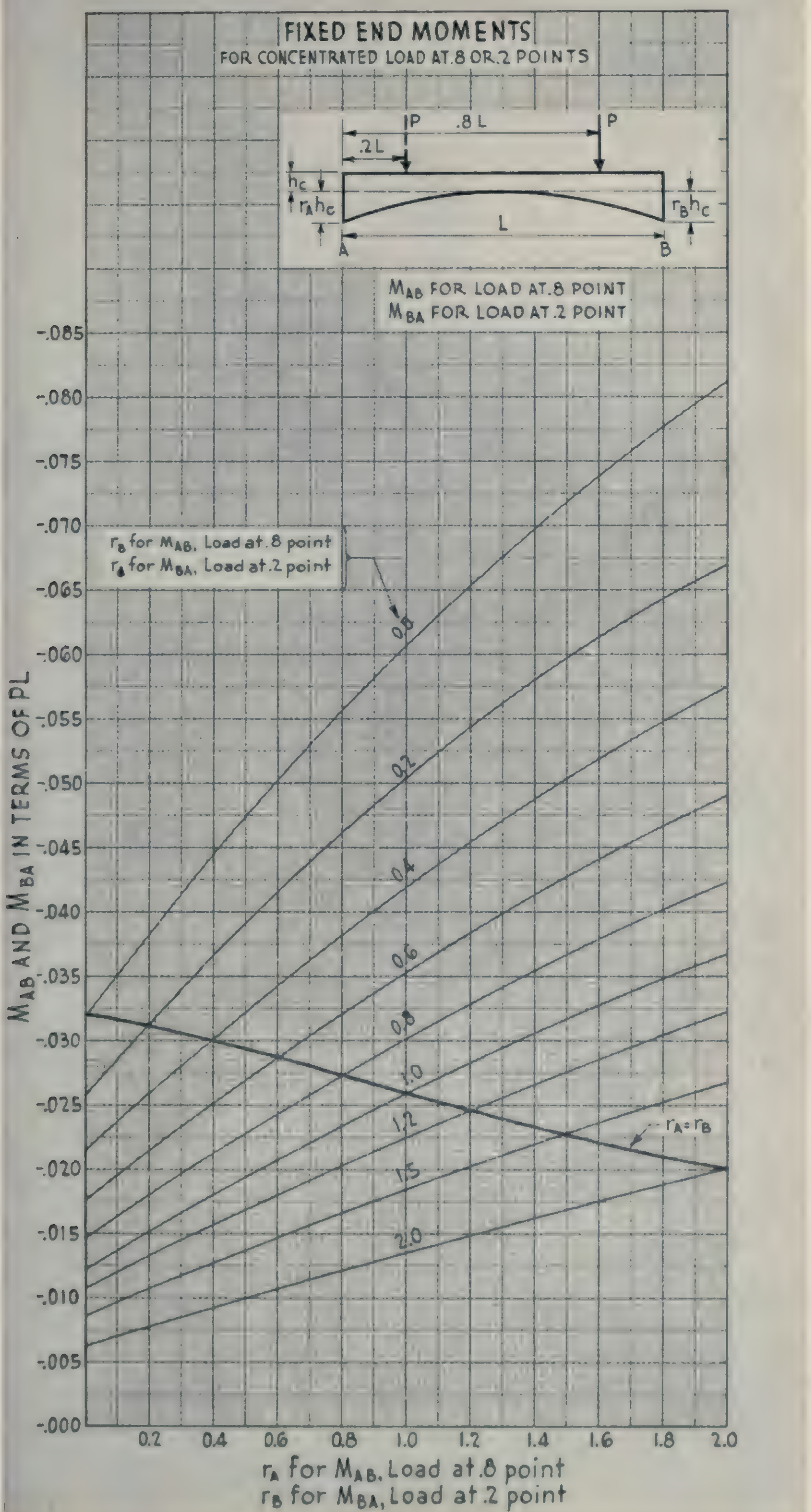

FIXED END MOMENTS
FOR CONCENTRATED LOAD AT .8 OR .2 POINTS
P
.8 L
P
.2L
h_c
$r_A h_c$
$r_B h_c$
L
A
B
M_{AB} FOR LOAD AT .8 POINT
M_{BA} FOR LOAD AT .2 POINT
r_B for M_{AB}, Load at .8 point
r_A for M_{BA}, Load at .2 point
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.5
2.0
$r_A = r_B$
M_{AB} AND M_{BA} IN TERMS OF PL
-.085
-.080
-.075
-.070
-.065
-.060
-.055
-.050
-.045
-.040
-.035
-.030
-.025
-.020
-.015
-.010
-.005
-.000
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
r_A for M_{AB}, Load at .8 point
r_B for M_{BA}, Load at .2 point

Fig. 10

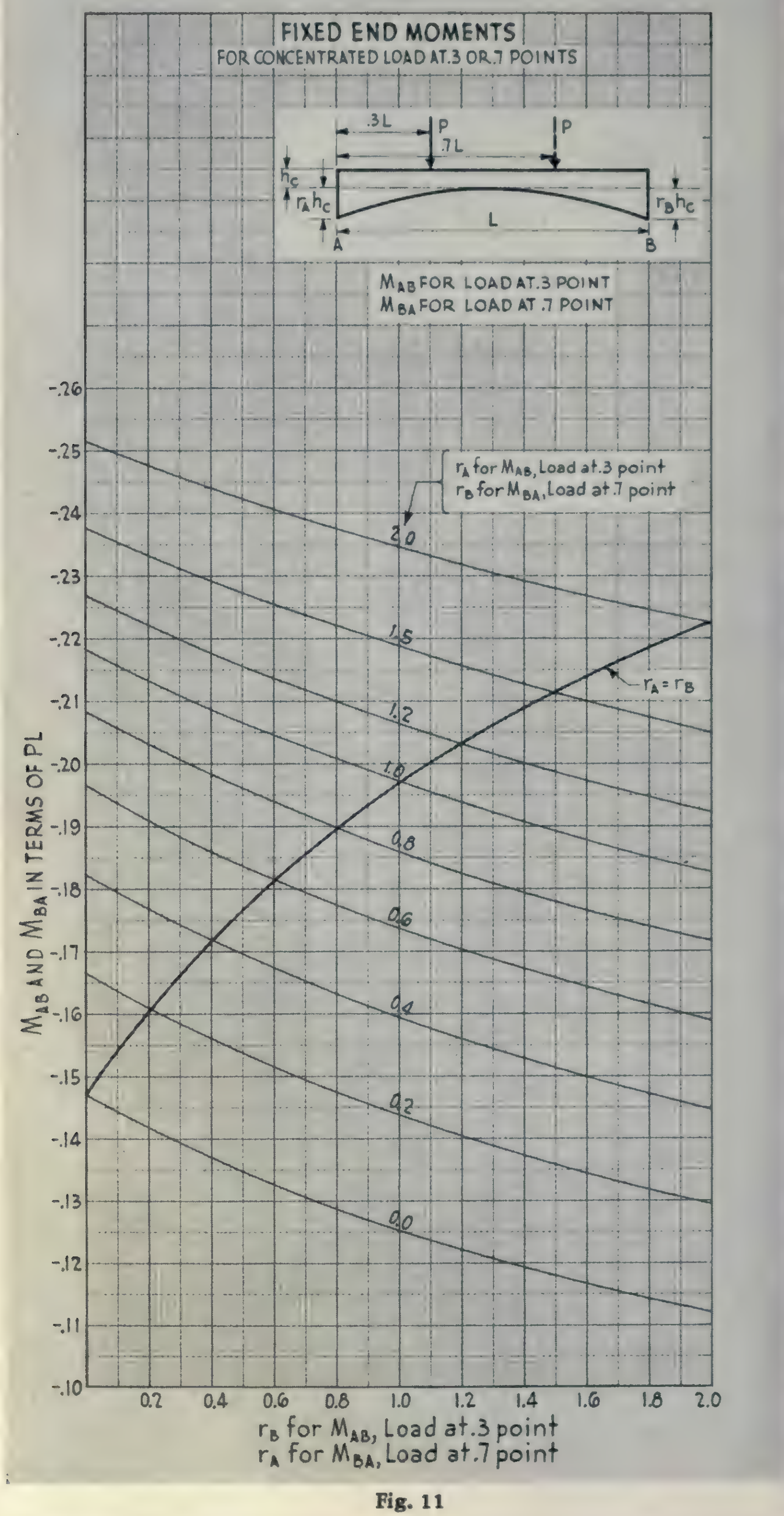

FIXED END MOMENTS
FOR CONCENTRATED LOAD AT .3 OR .7 POINTS
.3L
P
P
.7L
h_c
$r_A h_c$
$r_B h_c$
L
A
B
M_{AB} FOR LOAD AT .3 POINT
M_{BA} FOR LOAD AT .7 POINT
r_A for M_{AB}, Load at .3 point
r_B for M_{BA}, Load at .7 point
2.0
1.5
1.2
1.0
0.8
0.6
0.4
0.2
0.0
$r_A = r_B$
M_{AB} AND M_{BA} IN TERMS OF PL
-.26
-.25
-.24
-.23
-.22
-.21
-.20
-.19
-.18
-.17
-.16
-.15
-.14
-.13
-.12
-.11
-.10
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
r_B for M_{AB}, Load at .3 point
r_A for M_{BA}, Load at .7 point

Fig. 11

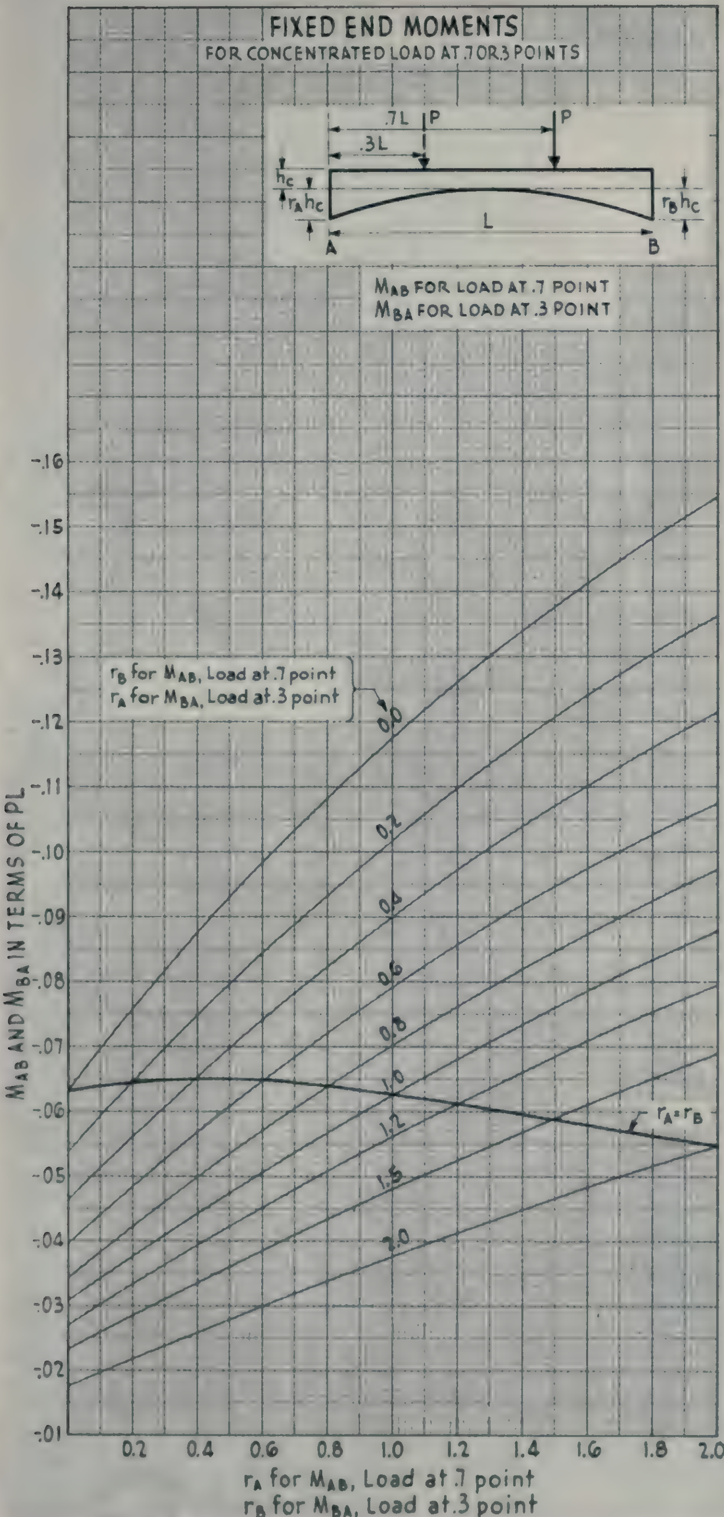

FIXED END MOMENTS
FOR CONCENTRATED LOAD AT .7 OR .3 POINTS
.7L
.3L
P
P
h_c
$r_A h_c$
$r_B h_c$
L
A
B
M_{AB} FOR LOAD AT .7 POINT
M_{BA} FOR LOAD AT .3 POINT
r_B for M_{AB}, Load at .7 point
r_A for M_{BA}, Load at .3 point
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.5
2.0
$r_A = r_B$
M_{AB} AND M_{BA} IN TERMS OF PL
-.16
-.15
-.14
-.13
-.12
-.11
-.10
-.09
-.08
-.07
-.06
-.05
-.04
-.03
-.02
-.01
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
r_A for M_{AB}, Load at .7 point
r_B for M_{BA}, Load at .3 point

Fig. 12

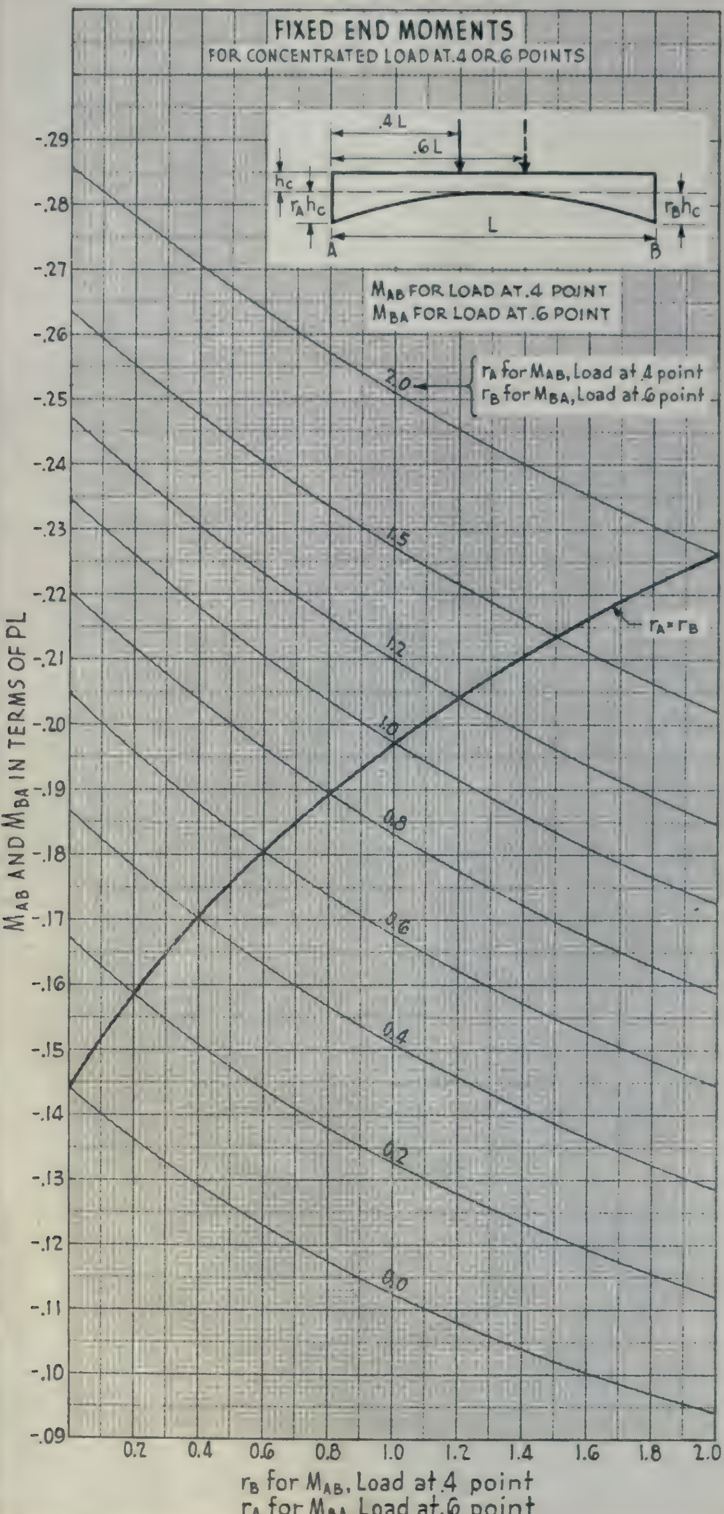

FIXED END MOMENTS
FOR CONCENTRATED LOAD AT .4 OR .6 POINTS
.4 L
.6 L
h_c
$r_A h_c$
$r_B h_c$
L
A
B
M_{AB} FOR LOAD AT .4 POINT
M_{BA} FOR LOAD AT .6 POINT
r_A for M_{AB}, Load at .4 point
r_B for M_{BA}, Load at .6 point
2.0
1.5
1.2
1.0
0.8
0.6
0.4
0.2
0.0
$r_A = r_B$
M_{AB} AND M_{BA} IN TERMS OF PL
-.29
-.28
-.27
-.26
-.25
-.24
-.23
-.22
-.21
-.20
-.19
-.18
-.17
-.16
-.15
-.14
-.13
-.12
-.11
-.10
-.09
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
r_B for M_{AB}, Load at .4 point
r_A for M_{BA}, Load at .6 point

Fig. 13

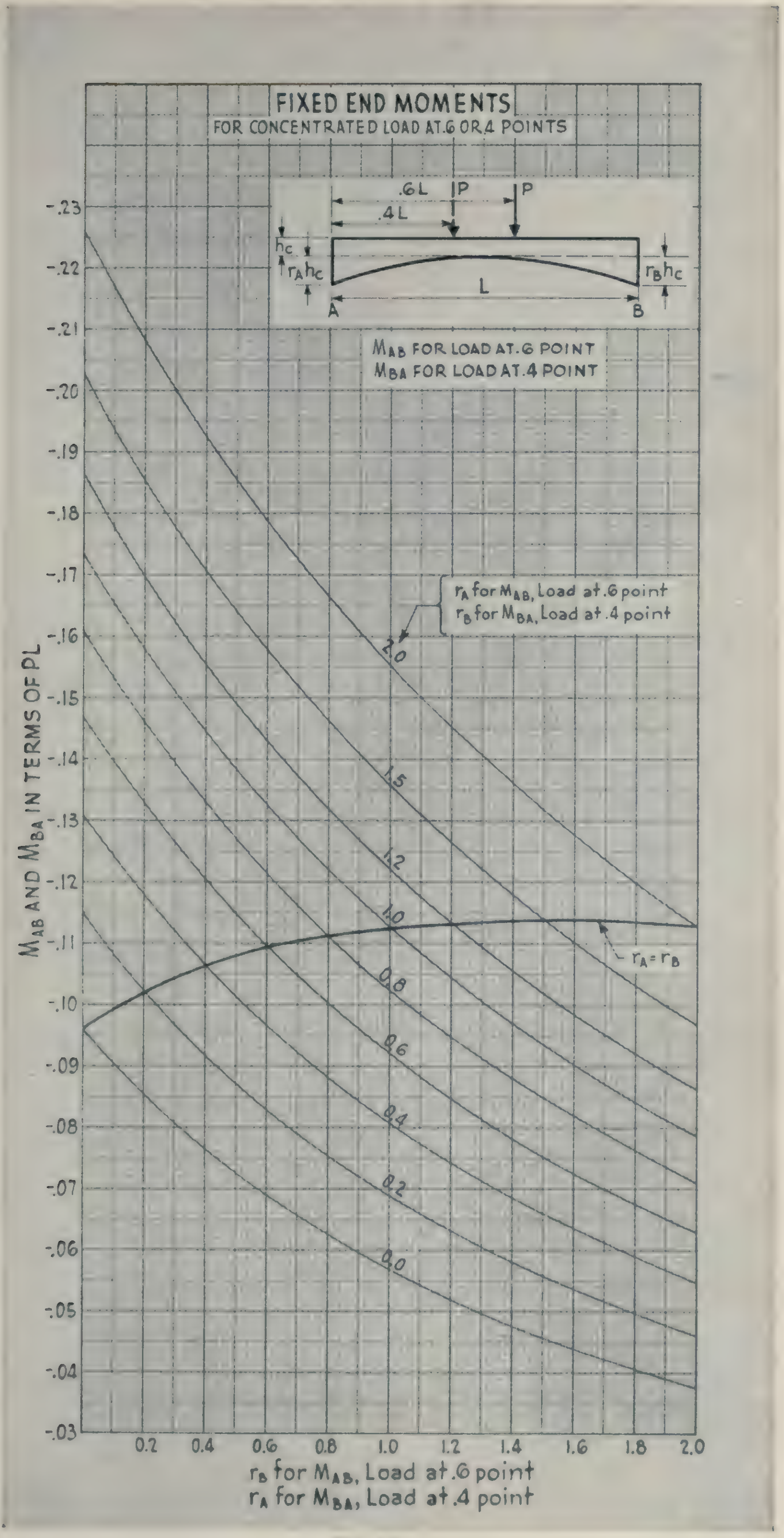

FIXED END MOMENTS
FOR CONCENTRATED LOAD AT .6 OR .4 POINTS
.6L
.4L
P
P
h_c
$r_A h_c$
$r_B h_c$
L
A
B
M_{AB} FOR LOAD AT .6 POINT
M_{BA} FOR LOAD AT .4 POINT
r_A for M_{AB}, Load at .6 point
r_B for M_{BA}, Load at .4 point
2.0
1.5
1.2
1.0
0.8
0.6
0.4
0.2
0.0
$r_A = r_B$
M_{AB} AND M_{BA} IN TERMS OF PL
-.23
-.22
-.21
-.20
-.19
-.18
-.17
-.16
-.15
-.14
-.13
-.12
-.11
-.10
-.09
-.08
-.07
-.06
-.05
-.04
-.03
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
r_B for M_{AB}, Load at .6 point
r_A for M_{BA}, Load at .4 point

Fig. 14

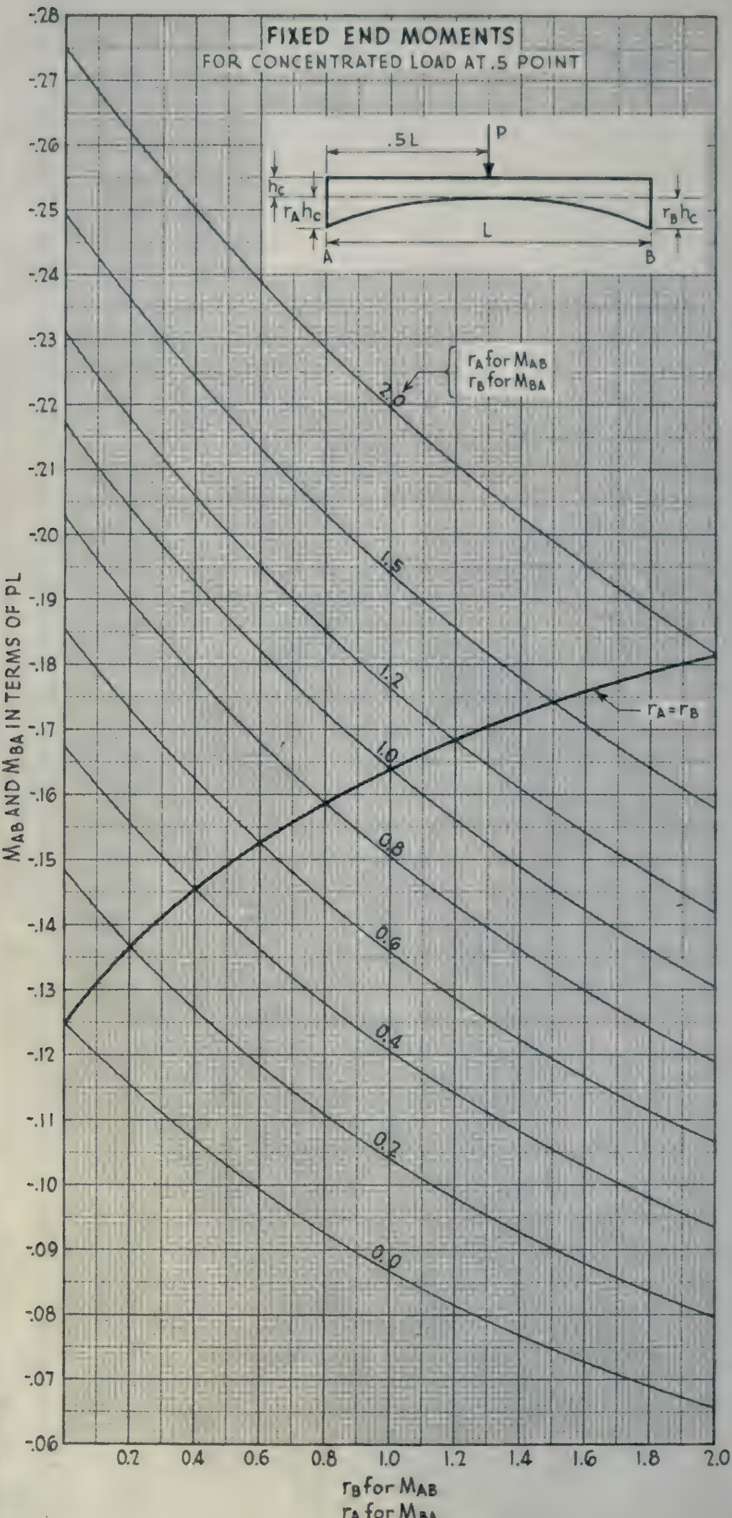

FIXED END MOMENTS
FOR CONCENTRATED LOAD AT .5 POINT
.5L
P
h_c
$r_A h_c$
$r_B h_c$
L
A
B
r_A for M_{AB}
r_B for M_{BA}
2.0
1.5
1.2
1.0
0.8
0.6
0.4
0.2
0.0
$r_A = r_B$
M_{AB} AND M_{BA} IN TERMS OF PL
-.28
-.27
-.26
-.25
-.24
-.23
-.22
-.21
-.20
-.19
-.18
-.17
-.16
-.15
-.14
-.13
-.12
-.11
-.10
-.09
-.08
-.07
-.06
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
r_B for M_{AB}
r_A for M_{BA}

Fig. 15

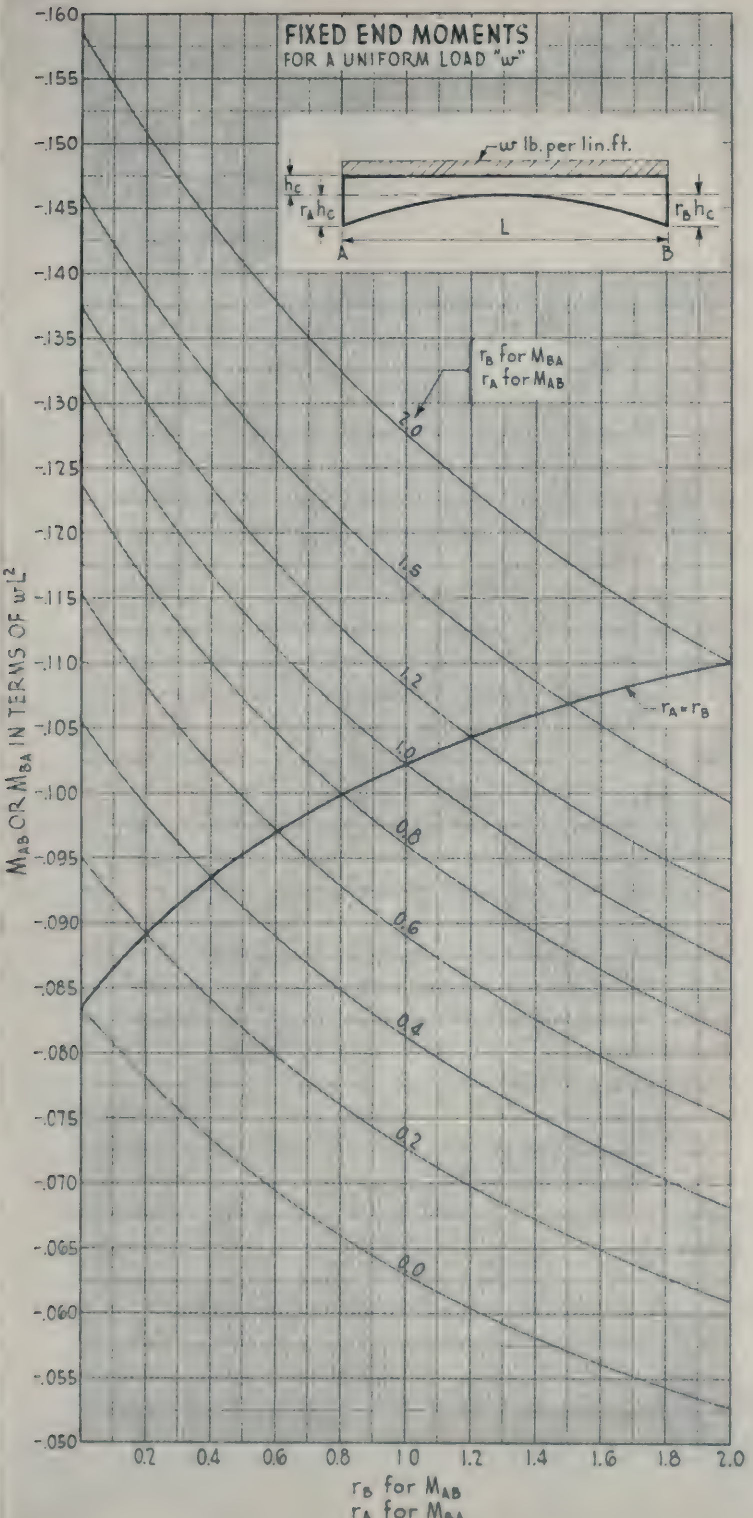

FIXED END MOMENTS
FOR A UNIFORM LOAD "w"
w lb. per lin. ft.
h_c
$r_A h_c$
$r_B h_c$
L
A
B
r_B for M_{BA}
r_A for M_{AB}
$r_A = r_B$
2.0
1.5
1.2
1.0
0.8
0.6
0.4
0.2
0.0
M_{AB} OR M_{BA} IN TERMS OF wL^2
-.160
-.155
-.150
-.145
-.140
-.135
-.130
-.125
-.120
-.115
-.110
-.105
-.100
-.095
-.090
-.085
-.080
-.075
-.070
-.065
-.060
-.055
-.050
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
r_B for M_{AB}
r_A for M_{BA}

Fig. 16

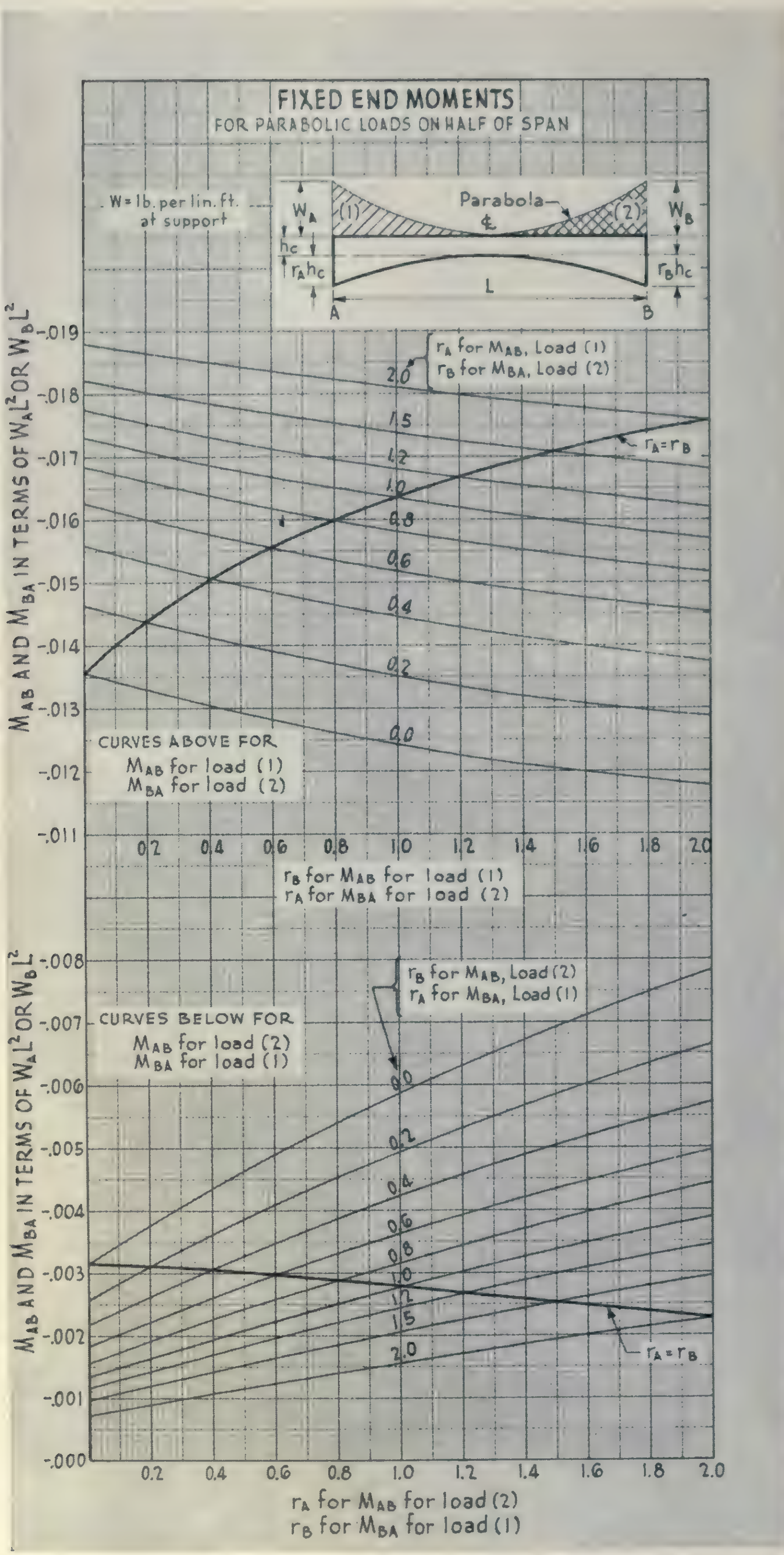

FIXED END MOMENTS
FOR PARABOLIC LOADS ON HALF OF SPAN
W = lb. per lin. ft. at support
Parabola
L
A
B
r_A for M_{AB}, Load (1)
r_B for M_{BA}, Load (2)
$r_A = r_B$
M_{AB} AND M_{BA} IN TERMS OF $W_A L^2$ OR $W_B L^2$
CURVES ABOVE FOR
M_{AB} for load (1)
M_{BA} for load (2)
r_B for M_{AB} for load (1)
r_A for M_{BA} for load (2)
r_B for M_{AB}, Load (2)
r_A for M_{BA}, Load (1)
CURVES BELOW FOR
M_{AB} for load (2)
M_{BA} for load (1)
r_A for M_{AB} for load (2)
r_B for M_{BA} for load (1)

Fig. 17

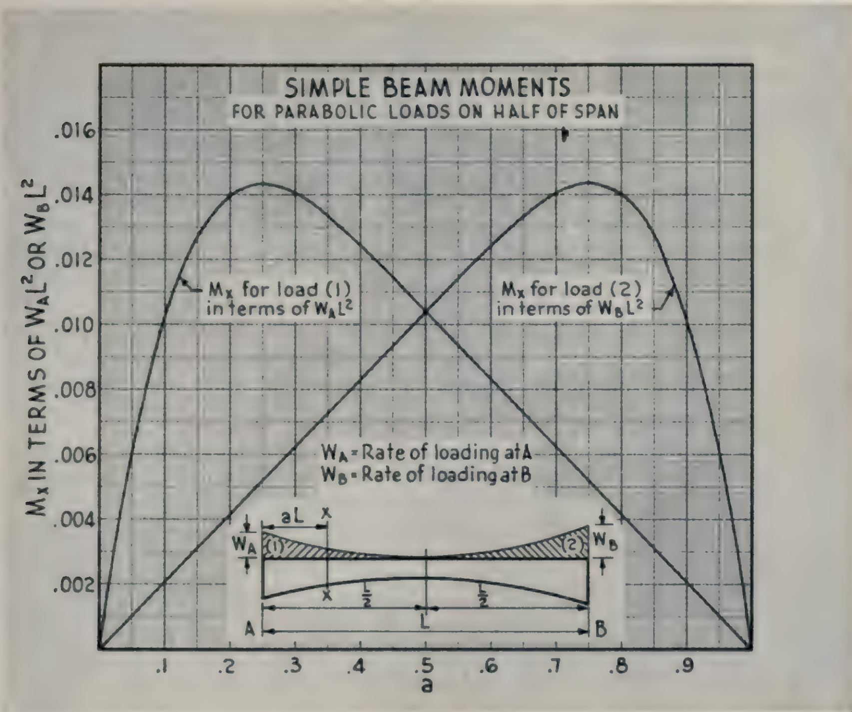


Fig. 17a

Accuracy of Formulas

The accuracy of the formulas in Table I is readily demonstrated by carrying through a few cycles of moment distribution in the usual manner. It can be shown that as the number of cycles is increased, the results obtained approach the values secured by means of the formulas as limits. This fact is apparent from a study of Tables III and IV, in which are tabulated the moments at supports for loads in each span of the three and four-span bridges used in Design Problems Nos. 1 and 3 obtained as the results of several cycles of moment distribution and by the formulas. It is, of course, unnecessary to carry out an actual design problem to as many significant figures as has been done in these tables for the purpose of demonstration.

TABLE III—Moments at Supports of Three-Span Bridge, Design Problem No. 1, Obtained by Moment Distribution and by Formulas in Table I

Number of cycles of distribution	Load in Span 1		Load in Span 2		Load in Span 3	
	M_B	M_C	M_B	M_C	M_B	M_C
4	$0.4775M_1$	$-0.1456M_1$	$0.5225M^F_{BC} + 0.1192M^F_{CB}$	$0.1378M^F_{BC} + 0.5753M^F_{CB}$	$-0.1192M_3$	$0.4247M_3$
5	$0.4759M_1$	$-0.1456M_1$	$0.5241M^F_{BC} + 0.1192M^F_{CB}$	$0.1378M^F_{BC} + 0.5771M^F_{CB}$	$-0.1192M_3$	$0.4247M_3$
6	$0.4758M_1$	$-0.1460M_1$	$0.5241M^F_{BC} + 0.1195M^F_{CB}$	$0.1462M^F_{BC} + 0.5771M^F_{CB}$	$-0.1195M_3$	$0.4228M_3$
By formula	$0.4757M_1$	$-0.1462M_1$	$0.5242M^F_{BC} + 0.1195M^F_{CB}$	$0.1462M^F_{BC} + 0.5775M^F_{CB}$	$-0.1195M_3$	$0.4227M_3$
	(Equation 22)	(Equation 23)	(Equation 24)	(Equation 25)	(Equation 26)	(Equation 27)

TABLE IV—Moments at Supports of Four-Span Bridge, Design Problem No. 3, Obtained by Moment Distribution and by Formulas in Table I

Number of cycles of distribution	Load in Span 1			Load in Span 2		
	M_B	M_C	M_D	M_B	M_C	M_D
5	$0.5471M_1$	$-0.2395M_1$	$0.0850M_1$	$0.4529M^F_{BC} + 0.1772M^F_{CB}$	$0.2395M^F_{BC} + 0.5000M^F_{CB}$	$-0.0850M^F_{BC} - 0.1772M^F_{CB}$
7	$0.5396M_1$	$-0.2395M_1$	$0.0924M_1$	$0.4604M^F_{BC} + 0.1927M^F_{CB}$	$0.2395M^F_{BC} + 0.5000M^F_{CB}$	$-0.0924M^F_{BC} - 0.1927M^F_{CB}$
9	$0.5370M_1$	$-0.2395M_1$	$0.0950M_1$	$0.4630M^F_{BC} + 0.1980M^F_{CB}$	$0.2395M^F_{BC} + 0.5000M^F_{CB}$	$-0.0950M^F_{BC} - 0.1980M^F_{CB}$
By formula	$0.5361M_1$	$-0.2395M_1$	$0.0964M_1$	$0.4645M^F_{BC} + 0.2010M^F_{CB}$	$0.2395M^F_{BC} + 0.5000M^F_{CB}$	$-0.0964M^F_{BC} - 0.2010M^F_{CB}$
	(Eq. 44)	(Equation 45)	(Eq. 46)	(Equation 47)	(Equation 48)	(Equation 49)

Moments Due to Change in Length of Deck

When the deck is unrestrained by the piers, in other words is freely supported throughout, changes in length of the deck due to temperature variations or changes in moisture content of the concrete (neglecting secondary stress) will not create any stress in the deck or the piers. In case the deck is made integral with two or more piers, however, bending moments will result at the integral joints due to the deflection of the tops of the piers with reference to their bottoms.

The amount of footing displacement (the tendency is usually to spread because of the longer interior spans) due to dead and live load is problematical and dependent upon such factors as relative span lengths, ratio of pier heights to span lengths, soil modulus and type of pier or footing. It is customary to determine the moment at the top of piers due to *deck* load as though no displacement of the footings takes place as this assumption will give the maximum pier moment, and, while the assumption of no footing displacement tends to reduce the positive moments in the deck, this practice is considered to be the most logical as the effect of a tolerable displacement is so slight as to be negligible.

When a pier is hinged at the bottom as in Fig. 18, which is the usual condition, the moment at the top due to a lateral displacement relative to

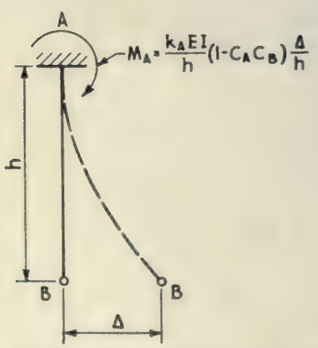


Fig. 18. Pier hinged at bottom.

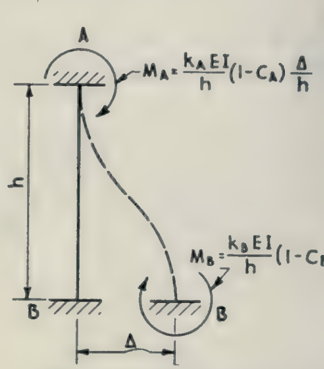


Fig. 19. Pier fixed at bottom.

the bottom, considering that the pier is temporarily fixed at the top, can be expressed as

$$M_A = \frac{k_A EI}{h}(1 - C_A C_B)\frac{\triangle}{h}^{*} \tag{12}$$

and when the pier is fixed at the bottom as in Fig. 19, the moment at the top will be

$$M_A = \frac{k_A EI}{h}(1 - C_A)\frac{\triangle}{h}^{*} \tag{13}$$

and at the bottom

$$M_B = \frac{k_B EI}{h}(1 - C_B)\frac{\triangle}{h}^{*} \tag{14}$$

in which

k_A and k_B = stiffness coefficients at top and bottom of pier, respectively
E = modulus of elasticity
I = moment of inertia at section of reference (*see second footnote page 15*)
C_A and C_B = carry-over factors at top and bottom of pier, respectively
$\triangle$ = relative displacement of top and bottom of pier due to change in deck length
h = height of pier

For a pier of uniform section hinged at the bottom the moment at the top becomes

$$M_A = \frac{3\,EI}{h^2}\triangle \tag{15}$$

which is simply the moment at the fixed end of a cantilever beam.

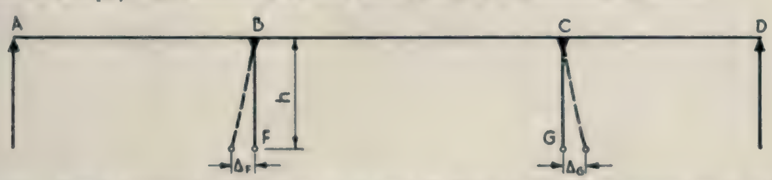


Fig. 20. Deck integral with piers at *B* and *C*. Effect of deck shortening is relative outward displacement of footings at *F* and *G*.

In the three-span structure, Fig. 20, the deck is freely supported at *A* and *D* and integral with piers *B* and *C*. If the piers are identical in all respects, the movements $\triangle_F$ and $\triangle_G$ will be the same and equal to one-half the total expansion or contraction of the deck between the two piers and the structure is not subject to sidesway. Having $\triangle_F$ and $\triangle_G$, M^F_{BF} and M^F_{CG} may be found directly from formulas shown with Figs. 18 and 19 and as given in Equations 12 to 15.

If the two piers are unlike, the deflections $\triangle_F$ and $\triangle_G$ will not be equal but will depend upon the stiffnesses of the piers and the structure will be subject to sidesway which will necessitate a slight correction in the final

*For derivation see *One-Story Concrete Frames Analyzed by Moment Distribution*, page 8, published by Portland Cement Association. The signs before the carry-over factors in Equations 12 and 13 are minus because carry-over factors are negative in this booklet. It is necessary to insert proper signs before M_A and M_B. If the sign of M_A is plus, the sign of M_B will be negative. See "Sign Convention", page 37.

moments*. It is not necessary to determine the deflection of each pier in order to compute the fixed end moments since they can be found in terms of the total change in deck length. When the piers are hinged at F and G,

$$M^F_{BF} = \frac{K_{BF}K_{CG}(1 - C_{BF}C_{FB})(1 - C_{CG}C_{GC})h_{BF}\triangle_T}{K_{BF}(1 - C_{BF}C_{FB})h_{CG} + K_{CG}(1 - C_{CG}C_{GC})h^2_{BF}} \tag{16}$$

and

$$M^F_{CG} = \frac{h_{CG}}{h_{BF}}M^F_{BF} \tag{17}$$

When piers are fixed at F and G,

$$M^F_{BF} = \frac{K_{BF}K_{CG}(1 - C_{BF})(1 - C_{CG})h_{BF}\triangle_T}{K_{BF}(1 - C_{BF})h^2_{CG} + K_{CG}(1 - C_{CG})h^2_{BF}} \tag{18}$$

and

$$M^F_{CG} = \frac{h_{CG}}{h_{BF}}M^F_{BF} \tag{19}$$

$$M^F_{FB} = \frac{K_{FB}K_{GC}(1 - C_{FB})(1 - C_{GC})h_{BF}\triangle_T}{K_{FB}(1 - C_{FB})h^2_{CG} + K_{GC}(1 - C_{CG})h^2_{BF}} \tag{20}$$

and

$$M^F_{CG} = \frac{h_{CG}}{h_{BF}}M^F_{FB} \tag{21}$$

in which $\triangle_T$ = total change in deck length between piers BF and CG and all other terms are as previously defined.

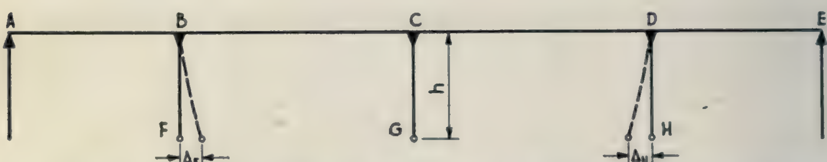


Fig. 21. Deck integral with piers at *B*, *C* and *D* in a structure symmetrical about *CG*. Effect of deck lengthening is relative inward displacement of footings at *F* and *H*.

In the four-span structure, Fig. 21, made integral at B, C, and D, the relative deflection of the piers B and D will be equal because the symmetry assumed requires that these two piers be identical and the structure is not subject to sidesway; also because of symmetry there would be no deflection at joint C regardless of how the center pier may differ from the other two.

In Table II, the fixed end moments can be evaluated by means of one of the Equations 12 to 21, depending upon the symmetry of the structure, the type of pier and the end conditions of the piers. Having the fixed end moments, the final distributed moments in the deck and piers at the joints can be obtained directly by means of the formulas in Table II assuming that sidesway is prevented.

For the purpose of design in the illustrative problems, the following

*Unless the piers are greatly different it will be sufficiently accurate to assume equal deflection and neglect sidesway. For a discussion of the effect of sidesway see *One-Story Concrete Frames Analyzed by Moment Distribution* published by Portland Cement Association.

values are assumed in order to determine change in deck length:

Coefficient of thermal expansion of concrete .000006
Coefficient of shrinkage of concrete .0002
Temperature range (F.) in northern climate +35 deg. and −45 deg.
Temperature range (F.) in southern climate +50 deg. and −30 deg.
Modulus of elasticity of concrete 3,000,000 p.s.i. (See page 77 for the value of E_c to be used in computations for deflections.)

Sign Convention

As is customary, a moment which causes tension on the top of the deck is considered negative and it will be convenient to give the same sign to a moment causing tension on the outside of the piers with reference to the centerline of the bridge when the piers are built integral with the deck. If this convention is followed and all fixed end moments due to deck load are considered negative and the fixed end moments at the tops of piers due to deck lengthening are negative, while those for deck shortening are positive, the signs of all final distributed moments will work out automatically by the formulas in Tables I and II. To obtain the total moment at any section the individual moments due to deck load and change in deck length are added algebraically.

Section V—Design Procedure for Slab Bridges

Reference should be made to Design Problem No. 1 when studying this section as it will help to clarify the different steps and show in detail how each step is carried out in practice.

1. Select r *Values*

It is not necessary to assume definite depth of slabs at the various sections; only the r values or ratios of increase in depth at supports to depth at the centerlines of spans need be assumed. Actual dimensions will be found later by a few simple substitutions.

Based upon span length ratios suggested in Section II—Layout, and an allowable $f_c = 0.45\,f'_c$ at supports compared with $f_c = 0.40\,f'_c$ at the centers of spans, the following r values are recommended:

When the end span is 35 ft. or less; $r = 0$ for all spans.
When the end span is between 35 and 50 ft.:

$r = 0$ to 0.4 at outer end of end spans
$= 0.4$ at first interior support
$= 0.5$ at all other supports

Usually it will be most economical to make $r = 0$ at the outer end of end spans regardless of length. When two or more continuous units of several spans each are required, the free ends abutting will generally be haunched for appearance; also when solid abutments are used, haunches at abutments contribute to appearance.

2. *Draw Influence Lines for Moments at Supports*

Having selected values for the parameters r_{AB}, r_{BA}, r_{BC}, etc., enter Fig. 5 and obtain corresponding carry-over factors, C_{AB}, C_{BA}, C_{BC}, etc.; then, with the same parameters, enter Fig. 6 and take off the stiffness coefficients, k_{AB}, k_{BA}, k_{BC}, etc., and if any piers are made integral with the deck, find their stiffness coefficients at junction of deck; if pier section is constant* the stiffness can be found from Fig. 6 taking $r_A = r_B = 0$.

By substituting stiffness coefficients in Equation 10 at the respective supports, the distribution factors D_{BA}, D_{BC}, D_{CB}, etc., may be evaluated. However, since the outer ends of the end spans of all continuous units are freely supported, it will be necessary to modify the stiffness of these spans at the first interior supports as shown in Equation 11 before use in determining distribution factors at these points.

Final moments at supports due to a unit load in any position in the various spans can now be obtained by substituting the proper carry-over and distribution factors in the formulas of Table I. The required fixed end moments necessary for substitution in the formulas can be taken from Figs. 7 to 17. By placing a concentrated load P at successive points along each span, ordinates for influence lines for the moments at each support will be obtained. For convenience, in the examples that follow, the load P was placed at consecutive tenth points, and the resulting moments tabulated as shown in Table V, page 43. With the ordinates thus determined, influence lines for M_B and M_C can be drawn as illustrated in Figs. 23 and 24 of the problem that follows.

3. *Maximum Live Load Moments at Critical Sections*

Maximum moments under truck-train loading must be determined at the supports, at centerlines of intermediate spans and at approximately the three-eighth point of the end spans. The exact point of maximum combined dead and live load positive moments in the end spans can be determined only by trial, so the live load moment at several sections near the anticipated critical section must be determined and later combined with the dead load moment. Two or more truck trains going in the same direction may have to be used to produce maximum moment, depending upon the critical section being investigated. The A.A.S.H.O. truck-train loading used in the examples is shown in Fig. 3.

Maximum negative moments at supports will be obtained when the spans adjacent to the support under consideration are loaded and also alternate spans beyond the two adjacent spans. For maximum positive moments within any span, the span under consideration should be loaded and also every alternate span. These loading schemes for maximum moments are shown as they are used in the illustrative problems that follow. For certain span lengths the critical position of loading may cause one wheel of a truck to be off the structure while the other is on or may place one wheel in an adjoining span not normally loaded for maximum moment at the particular section under consideration.

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*For tapered piers see *Handbook of Frame Constants*, published by Portland Cement Association.

As illustrated in the design problems the desired moments are obtained by multiplying the ordinates to the influence lines under the loads by the respective loads. The final moment at the section being investigated will be the combined effect of all the loads on the structure.

4. *Dead Load Moments*

The dead load moments must be computed at the same points as the live load moments. Use will again be made of the formulas in Table I to determine the moments at supports by substituting in them the fixed end moments due to the dead load in each span. In an actual design the coefficients of the fixed end moments computed for live load under Step 3 can be used with the fixed end moments due to dead load because these coefficients depend upon the beam constants and not on the type of loading.

Having determined the negative moments at supports, the simple beam positive moments at the points under consideration may be found in the usual manner and from them the effect of the end moments at the points must be subtracted to give the desired positive moments. It will be convenient to determine the dead load moments in terms of a uniform load w* over the whole bridge and W_{BA}, W_{BC}, etc., representing the weight per linear foot of the parabolic haunches based on the cross sections at the supports. Fixed end moments for uniform load and the haunch loads can be taken from Figs. 16 and 17.

5. *Assume Depth at Center of Span*

A trial value of h_c can now be assumed. The center depth is dependent upon length of spans and whether or not the slab is haunched. The value of h_c for slabs of constant depth throughout, for the assumed stresses and loading, will be about 1/32 of the length of the longest intermediate span and 1/35 to 1/40 for haunched slabs.

The actual dead load moments can now be determined by substituting the values of w, W_{BA}, W_{BC}, etc., corresponding to the assumed value of h_c in the expressions for moment found in Step 4. These moments at the critical sections must be added to the live load moments and the required depth determined to see if it agrees with the assumed depth h_c. If not, assume new h_c and repeat. In the case of slabs of uniform depth, only the section subject to maximum moment need be investigated. If slab is haunched, depth at support will be $h_c\ (1+r)$.

6. *Draw Maximum Moment Curves*

After the required slab depth has been determined at critical points under Step 5, curves of maximum moments can be drawn. To do this locate closely the point of maximum positive moment in end spans; find maximum positive and negative moments at 0.7 point of end spans and at 0.2 and

*This assumes h_c at centerline of all spans the same; if different, all centerline depths and hence the w for each span can be expressed as ratios of one of them.

0.8 or 0.3 and 0.7 points of intermediate spans. These values, in addition to those found under Steps 3 and 4, will make it possible to draw moment curves sufficiently accurate for design purposes by passing parabolas through the points of positive moment and straight lines through the points of negative moments.

7. *Shear*

Shear is not likely to influence the required depth h_c, but should always be checked. If sections at supports should prove inadequate for shear, use haunches or if haunches are already provided, increase them. This increase in haunch will allow a corresponding decrease in h_c and will necessitate repeating Step 5.

8. *Moment in Piers Due to Deck Load*

Maximum moment is produced at the top of a pier that is integral with the deck when the difference between the moments in the deck adjacent to the pier is maximum. This condition will result when the longer span is loaded to produce maximum negative moment and the shorter span is not loaded.

It is usually desirable to reinforce both piers the same, and symmetrically about their axes, so it is only necessary to investigate the pier in which the moment will be maximum.

9. *Moments Due to Change in Length of Deck*

As a final step when deck and piers are integral, moments must be determined at critical sections by means of the formulas in Table II and

Continuous bridges with wide center spans and open abutments are especially well suited to rail-highway grade separations, as illustrated by De Baliviere Avenue Bridge over Wabash Railroad, St. Louis, Mo. Designed by Sverdrup and Parcel, consulting engineers, St. Louis, for Missouri State Highway Commission.

Webber Creek Bridge near Placerville, Calif., is of the continuous girder type in which deck and open frame bents are integral. Three interior spans are each 71 ft. and the end spans are 52 ft. Designed by California State Division of Highways. F. W. Panhorst, bridge engineer.

as discussed under the heading "Moments Due to Change in Length of Deck"—Section IV. These moments must be combined with moments due to dead and live loads as illustrated in Design Problem No. 2. It is customary to allow 25 per cent or more increase in stress when the effect of temperature change is included.

DESIGN PROBLEM No. 1

The three-span slab bridge of Layout Problem No. 1 will be used to illustrate the design procedure just discussed. It has purposely been made unsymmetrical to make the problem more general. (*See Fig. 22.*)

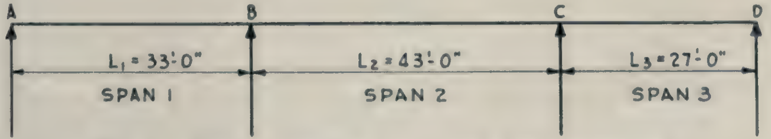


Fig. 22. Design Problem No. 1, deck freely supported.

Assume A.A.S.H.O. H-15 truck-train loading plus impact; 10-ft. traffic lanes*; f_c = 1,200 p.s.i. for positive moments and 1,350 p.s.i. for negative moments; f_s = 18,000 p.s.i.

*The 10-ft. traffic lane has been adopted by the A.A.S.H.O. and is the basis for load distribution in its 1949 *Standard Specifications for Highway Bridges*. The actual effective traffic lanes are even greater. See *Public Roads*, September, 1937, page 129.

Step 1. Select r *Values*

The r values for all spans will be taken equal to 0.

Step 2. Draw Influence Lines for Moment at Supports

Since $r = 0$ at each support, all carry-over factors, C, will be equal and all stiffness coefficients, k, will be the same. From Figs. 5 and 6:

$$C_{AB} = C_{BA} = C_{BC} \quad . \quad . \quad . \quad . \quad = -0.5$$

$$k_{AB} = k_{BA} = k_{BC} \quad . \quad . \quad . \quad . \quad . \quad = \quad 4.0$$

Compute distribution factors:

$$D_{BA} = \frac{\dfrac{(1-0.5 \times 0.5)4^*}{33}}{\dfrac{0.75 \times 4^*}{33} + \dfrac{4}{43}} = 0.494; \text{ and } D_{BC} = 1-0.494 = 0.506$$

(by Equation 10)

Similarly $D_{CB} = 0.456$; $D_{CD} = 0.544$

Next compute moments at each interior support as a concentrated load moves across the bridge. Since the deck is not integral with supports in this problem, $M_{BA} = M_{BC}$ and $M_{CB} = M_{CD}$ for all conditions of loading, so support moments may be simply denoted M_B and M_C respectively.

Load in Span 1

Find M_B and M_C by use of formulas in Table I.

$$M_B = \frac{1 - D_{BA} - U}{1 - U} M_1 \qquad U = C_{BC}C_{CB}D_{BC}D_{CB}$$

$$U = -0.5 \times (-0.5) \times 0.506 \times 0.456 = 0.058$$

$$M_B = \frac{1 - 0.494 - 0.058}{1 - 0.058} M_1 = 0.476\, M_1 \tag{22}$$

$$M_C = \frac{V}{1 - U} M_1 \qquad V = C_{BC}D_{BC}D_{CD}$$

$$V = -0.5 \times 0.506 \times 0.544 = -0.138$$

$$M_C = \frac{-0.138}{0.942} M_1 = -0.146\, M_1 \tag{23}$$

---

*Since end A is freely supported, stiffness k_{BA} is modified by $(1 - C_{AB}C_{BA})$.

Load in Span 2

$$M_B = \frac{D_{BA} M^F_{BC} - W M^F_{CB}}{1 - U} \qquad W = C_{CB} D_{CB} D_{BA}$$

$$W = -0.5 \times 0.456 \times 0.494 = -0.113$$

$$M_B = \frac{0.494\ M^F_{BC} + 0.113\ M^F_{CB}}{0.942} = 0.524\ M^F_{BC} + 0.120\ M^F_{CB} \qquad (24)$$

$$M_C = \frac{D_{CD} M^F_{CB} - V M^F_{BC}}{1 - U} = \frac{0.544\ M^F_{CB} + 0.138\ M^F_{BC}}{0.942}$$

$$= 0.578\ M^F_{CB} + 0.146\ M^F_{BC} \qquad (25)$$

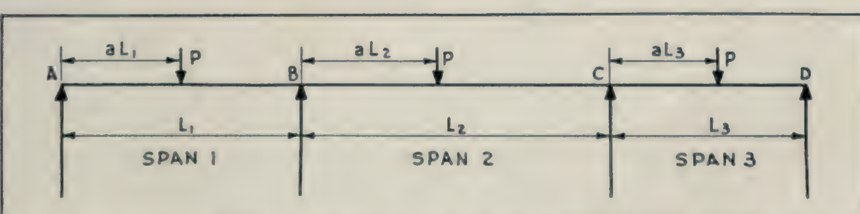
Load in Span 3

$$M_B = \frac{W}{1 - U} M_3 = \frac{-0.113}{0.942} M_3 = -0.120 M_3 \qquad (26)$$

$$M_C = \frac{1 - D_{CD} - U}{1 - U} M_3 = \frac{1 - 0.544 - 0.058}{0.942} M_3 = 0.423 M_3 \quad (27)$$

Having the expressions for moment at the supports (Equations 22 to 27) for a load at any position it is only necessary to substitute the fixed end moments due to a load P to obtain ordinates to influence lines which are recorded in Table V. Influence lines for M_B and M_C can now be drawn. (*See Figs. 23 and 24.*)

Table V—Ordinates for Influence Lines for Moments at Supports



a	M^F_{AB}* (Figs. 7-15)	M^F_{BA}* (Figs. 7-15)	M_1 (Eq. 1 pg. 17)	M_3 (Eq. 3 pg. 17)	Load in Span 1		Load in Span 2		Load in Span 3	
					M_B (Eq. 22)	M_C (Eq. 23)	M_B (Eq. 24)	M_C (Eq. 25)	M_B (Eq. 26)	M_C (Eq. 27)
.1	−.081 PL	−.009 PL	−.050 PL_1	−.086 PL	−.024 PL_1	+.007 PL_1	−.044 PL_2	−.017 PL_2	+.010 PL_3	−.036 PL_3
.2	−.128 "	−.032 "	−.096 "	−.144 "	−.046 "	+.014 "	−.071 "	−.037 "	+.017 "	−.061 "
.3	−.147 "	−.063 "	−.137 "	−.179 "	−.065 "	+.020 "	−.085 "	−.058 "	+.021 "	−.076 "
.4	−.144 "	−.096 "	−.168 "	−.192 "	−.080 "	+.025 "	−.087 "	−.077 "	+.023 "	−.081 "
.5	−.125 "	−.125 "	−.188 "	−.188 "	−.089 "	+.027 "	−.081 "	−.091 "	+.023 "	−.080 "
.6	−.096 "	−.144 "	−.192 "	−.168 "	−.091 "	+.028 "	−.068 "	−.097 "	+.020 "	−.071 "
.7	−.063 "	−.147 "	−.179 "	−.137 "	−.085 "	+.026 "	−.051 "	−.094 "	+.016 "	−.058 "
.8	−.032 "	−.128 "	−.144 "	−.096 "	−.069 "	+.021 "	−.032 "	−.079 "	+.012 "	−.041 "
.9	−.009 "	−.081 "	−.086 "	−.050 "	−.041 "	+.013 "	−.014 "	−.048 "	+.006 "	−.021 "

*Subscripts AB and BA correspond to subscripts in Figs. 7-15 and the moments are the fixed end moments at the left and right ends respectively of Spans 1, 2 and 3 when the proper values of L are used.

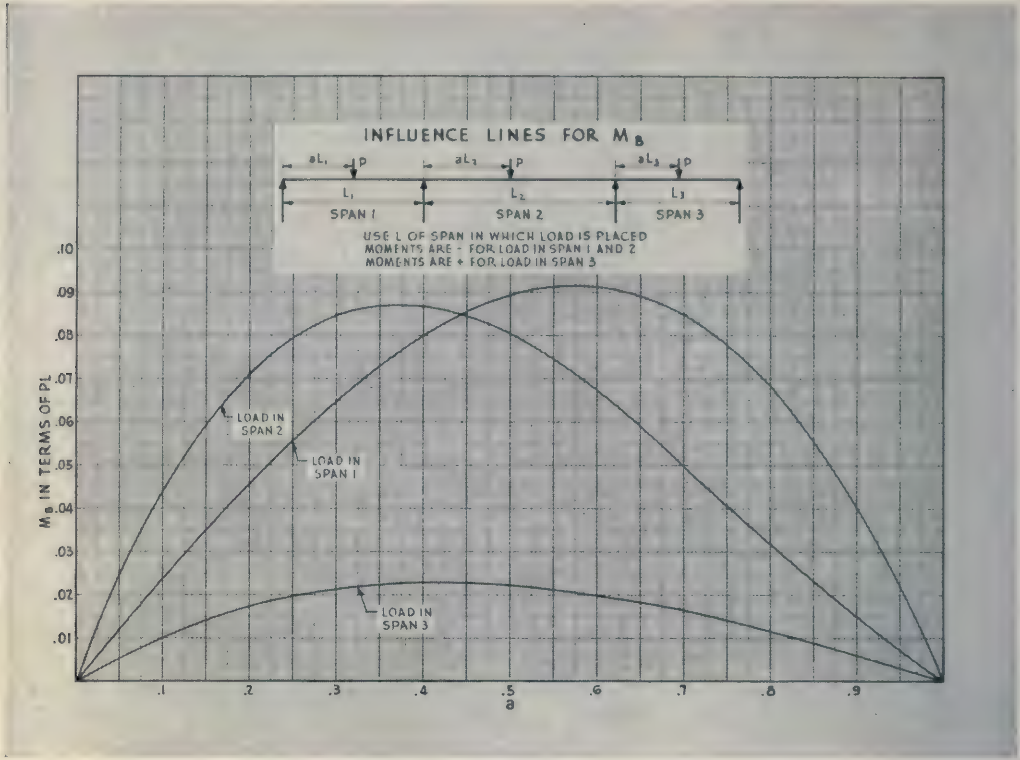

INFLUENCE LINES FOR M_B
aL_1 P
aL_2 P
aL_3 P
L_1
L_2
L_3
SPAN 1
SPAN 2
SPAN 3
USE L OF SPAN IN WHICH LOAD IS PLACED
MOMENTS ARE - FOR LOAD IN SPAN 1 AND 2
MOMENTS ARE + FOR LOAD IN SPAN 3
M_B IN TERMS OF PL
.10
.09
.08
.07
.06
.05
.04
.03
.02
.01
LOAD IN SPAN 2
LOAD IN SPAN 1
LOAD IN SPAN 3
.1
.2
.3
.4
.5
.6
.7
.8
.9
a

Fig. 23

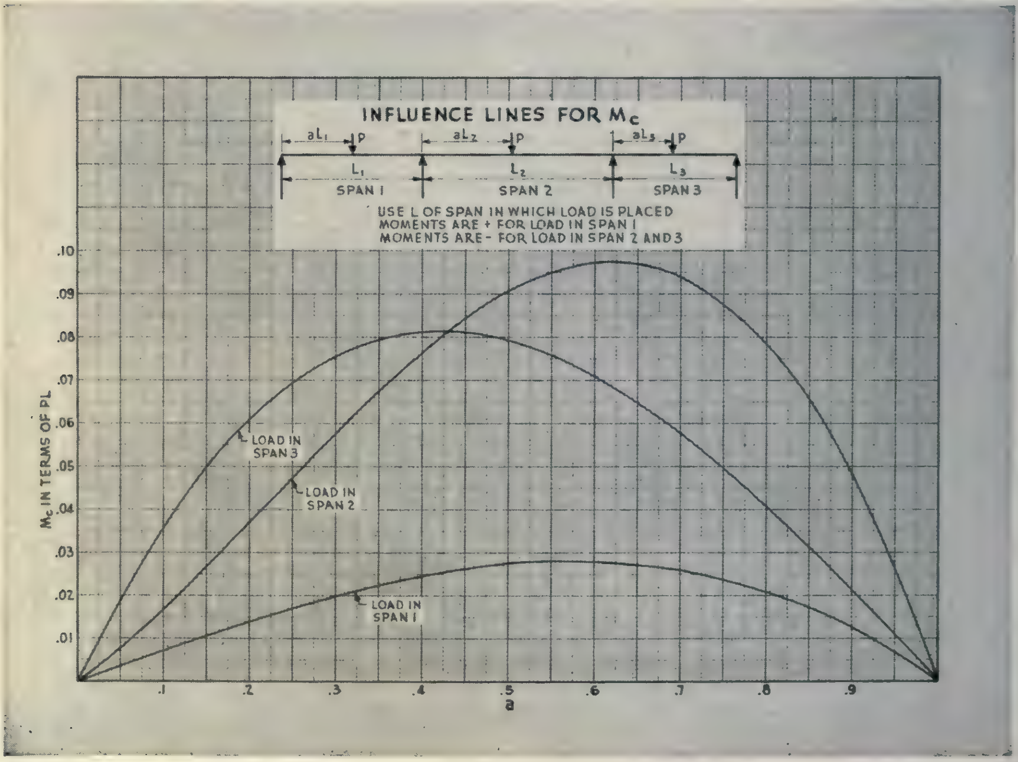

INFLUENCE LINES FOR M_C
aL_1 P
aL_2 P
aL_3 P
L_1
L_2
L_3
SPAN 1
SPAN 2
SPAN 3
USE L OF SPAN IN WHICH LOAD IS PLACED
MOMENTS ARE + FOR LOAD IN SPAN 1
MOMENTS ARE - FOR LOAD IN SPAN 2 AND 3
M_C IN TERMS OF PL
.10
.09
.08
.07
.06
.05
.04
.03
.02
.01
LOAD IN SPAN 3
LOAD IN SPAN 2
LOAD IN SPAN 1
.1
.2
.3
.4
.5
.6
.7
.8
.9
a

Fig. 24

Step 3. Maximum Live Load Moments at Critical Sections

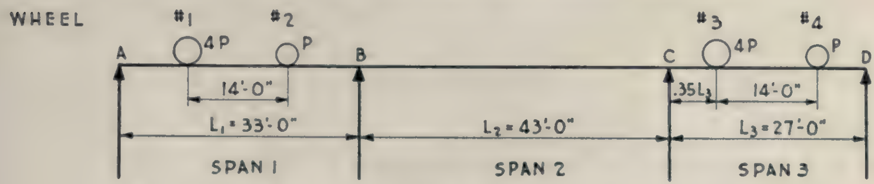


Fig. 25. Loading for maximum positive moment in Span 1.

For maximum positive moment in Span 1, first place loads in Span 3 so as to produce maximum $+M_B$, which is found to be when Wheel 3 is at $0.35L_3$, as shown in Fig. 25. From influence line for moment at B for load in Span 3, Fig. 23,

$$M_B = (0.023 \times 4P + 0.008 \times P)\ 27 = 2.70\ P \tag{28}$$

For Wheel 1 at $0.35L_1$ and Wheel 2 at $0.77L_1$ in Span 1

$$M_B = (-0.073 \times 4P - 0.074 \times P)\ 33 = -12.08\text{P} \tag{29}$$

and the positive moment under Wheel 1 is

$$\text{M}_{.35L_1} = 2.83P \times 0.35 \times 33 - 0.35\ (12.08 - 2.70)P = 29.4P \tag{30}$$

Similarly when Wheel 1 is at $0.38L_1$ and at $0.40L_1$

$$M_{.38L_1} = 29.9P \tag{31}$$

$$M_{.40L_1} = 30.0P \tag{32}$$

According to the A.A.S.H.O. Specifications:

$$\text{Impact factor} = \frac{50}{125 + 33 + 27} = 0.27 \therefore P = \frac{1.27 \times 3{,}000}{6^*} = 635 \text{ lb.}$$

and

$$M_{.35L_1} = 18{,}700 \text{ ft.lb.} \tag{33}$$

$$M_{.38L_1} = 19{,}000 \text{ ft.lb.} \tag{34}$$

$$M_{.40L_1} = 19{,}100 \text{ ft.lb.} \tag{35}$$

These values will be combined with the dead load moments to determine maximum design moments.

For maximum negative moment at B place loads in Spans 1 and 2. After a few movements of the truck train, moving from left to right, the position

*P per foot width is taken as the load of one front wheel, plus impact, distributed over a 6-ft. width of slab. The impact factor is dependent upon the length of the loaded spans so theoretically the value of P will vary slightly depending upon the spans loaded but little error would result if an average value were used.

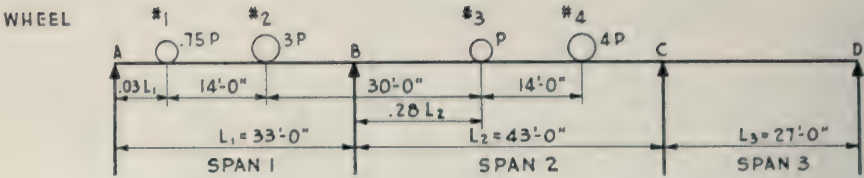


Fig. 26. Loading for maximum negative moment at support B.

indicated in Fig. 26 will be found to give maximum moment. It is usually necessary to repeat this process with truck train headed in the opposite direction, so as to insure securing the absolute maximum.

From the influence lines in Fig. 23:

$$\text{Max. } M_B = (-0.007 \times 0.75P - 0.086 \times 3P)\ 33 + (-0.083P - 0.066 \times 4P)\ 43$$
$$= -23.6P = -23.6 \times 625^{*} = -14{,}800 \text{ ft.lb.} \qquad (36)$$

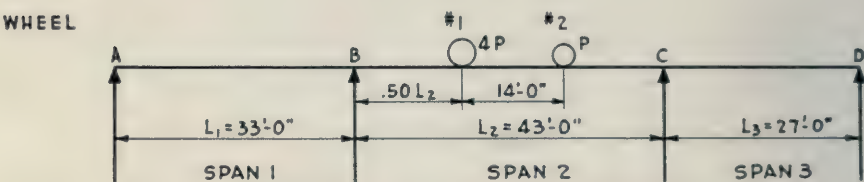


Fig. 27. Loading for maximum positive moment in Span 2.

For maximum positive moment in Span 2, Wheel 1 is placed at the center of the span as shown in Fig. 27. Actually the point of absolute maximum positive live load moment will be slightly off the centerline, but when the dead load moment is added to the total moment in this problem, the moments will be maximum at the center so it will be necessary to compute only the live load moment at that point.

$$M_{.50L_2} = \text{simple beam moment} + 0.5\ (M_B + M_C)$$
$$= (0.50 \times 4P + 0.17P)\ 0.5 \times 43$$
$$+ 0.5\,(-0.081 \times 4P - 0.027P - 0.091 \times 4P - 0.071P)43$$
$$= 29.7P$$
$$= 29.7 \times 649 = 19{,}300 \text{ ft.lb.} \qquad (37)$$

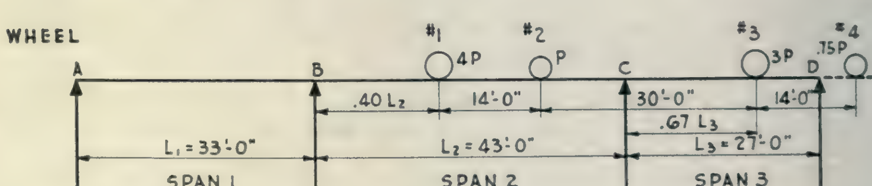


Fig. 28. Loading for maximum negative moment at support C.

*See footnote page 45.

In a manner similar to the above the maximum negative moment at C and the maximum positive moment at several points in Span 3 may be computed as:

$$M_C = -\ 13{,}900 \text{ ft.lb. } (\textit{for loading see Fig. 28}) \tag{38}$$

$$M_{.58L_3} = 16{,}000 \text{ ft.lb. } (\textit{for loading see Fig. 29}) \tag{39}$$

$$M_{.60L_3} = 15{,}900 \text{ ft.lb. } \text{“ “ “ “ “} \tag{40}$$

$$M_{.63L_3} = 15{,}800 \text{ ft.lb. } \text{“ “ “ “ “} \tag{41}$$

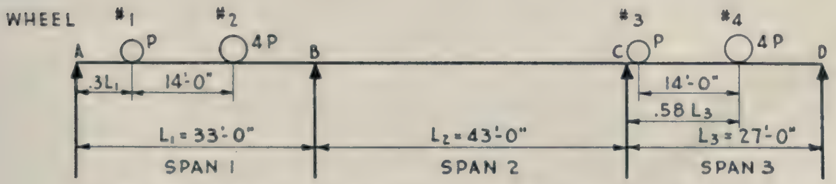


Fig. 29. Loading for maximum positive moment in Span 3.

Step 4. Dead Load Moments

Find moments M_B and M_C in terms of the yet undetermined uniform load w lb. per lin.ft. The coefficients of the fixed end moments are the same as those for concentrated loads under Step 2, therefore,

For load in Span 1:

$$M_B = 0.476M_1 = 0.476\ (-0.125w \times 33^2) = -\ \ 64.8w$$

For load in Span 2:

$$M_B = (0.524 + 0.120)\ M^{F*}_{BC} = 0.644\ (-0.083w \times 43^2) = -\ \ 98.8w$$

For load in Span 3:

$$M_B = -0.120M_3 = -0.120\ (-0.125w \times 27^2) = +\ \ \underline{10.9w}$$

$$\text{Total } M_B = -152.7w \tag{42}$$

For load in Span 1:

$$M_C = -0.146M_1 = -0.146\ (-0.125w \times 33^2) = +\ \ 19.9w$$

For load in Span 2:

$$M_C = (0.146 + 0.578)\ M^{F*}_{BC} = 0.724\ (-0.0833w \times 43^2) = -111.5w$$

For load in Span 3:

$$M_C = 0.423M_3 = 0.423\ (-0.125w \times 27^2) = -\ \ \underline{38.5w}$$

$$\text{Total } M_C = -130.1w \tag{43}$$

Step 5. Assume Depth at Center of Span

Assume $h_c = 16$ in. and check required depth at section of maximum combined dead and live load moment, which by observation will be at support B since the slab is of constant depth.

---

* $M^F_{BC} = M^F_{CB}$

$$M_B = -152.7 \times 200 - 14{,}800 = -45{,}300 \text{ ft.lb. } (\textit{see Equations 36 and 42})$$

$$\text{Required } h_c = 2 + \sqrt{\frac{45{,}300}{248^*}} = 15.5, \text{ say 16 in.}$$

Step 6. Draw Maximum Moment Curves

Since the assumed depth is satisfactory at support B it will be for all other sections so the total live and dead load moments may now be computed at the sections of maximum moment and at a sufficient number of other sections so moment curves can be drawn. (*See Figs. 30 to 33 for loading.*) The method of computation is similar to that already illustrated in detail

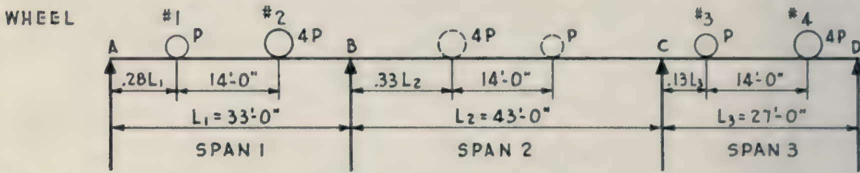


Fig. 30. Position of trucks in Spans 1 and 3 for maximum positive moment at $0.7L_1$, and position of truck in Span 2 for maximum negative moment at $0.7L_1$.

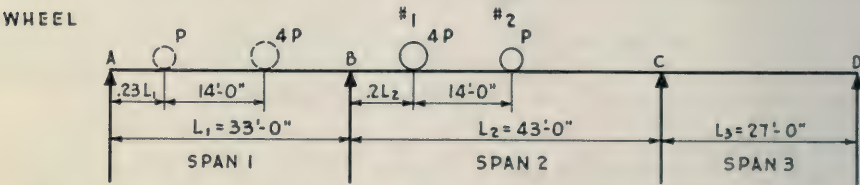


Fig. 31. Position of truck in Span 2 for maximum positive moment at $0.2L_2$, and position of truck in Span 1 for maximum negative moment at $0.2L_2$.

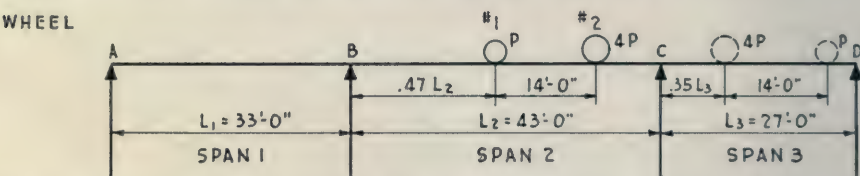


Fig. 32. Position of truck in Span 2 for maximum positive moment at $0.8L_2$, and position of truck in Span 3 for maximum negative moment at $0.8L_2$.

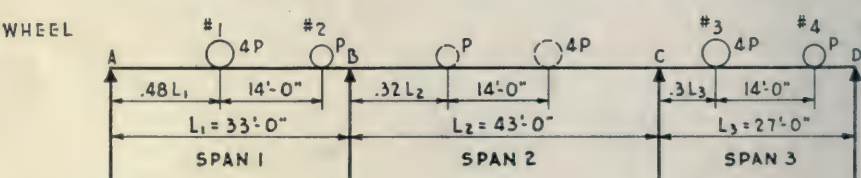


Fig. 33. Position of trucks in Spans 1 and 3 for maximum positive moment at $0.3L_3$, and position of truck in Span 2 for maximum negative moment at $0.3L_3$.

---

*$K = 248$ when $f_s = 18{,}000$ and $f_c = 0.45f'_c = 1{,}350$

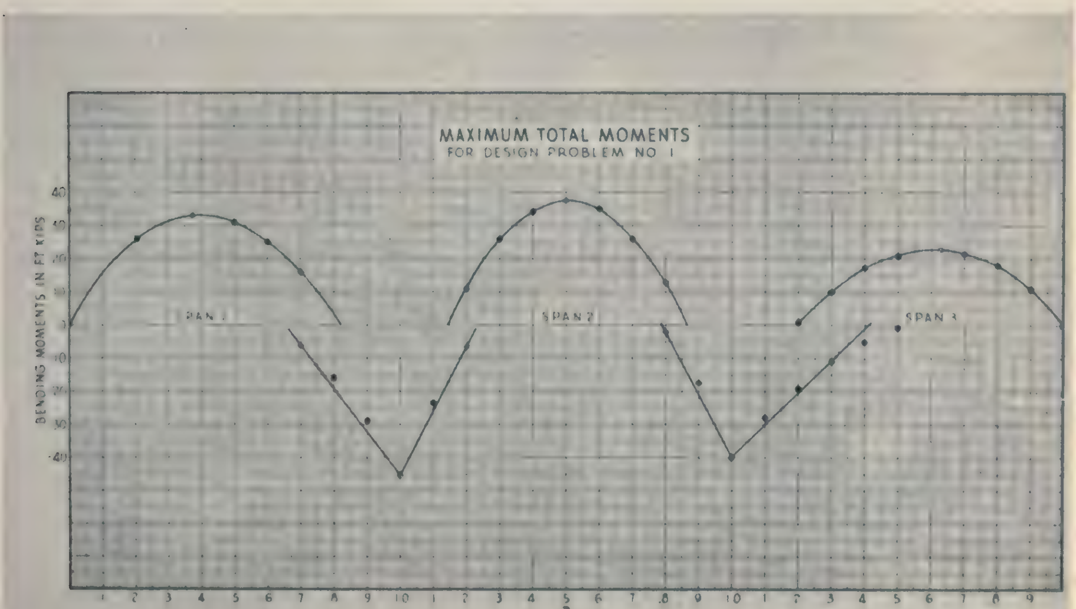
Table VI—Maximum Moments at Sections Required for Drawing Moment Curves

Span	Section	Dead load moment ft.lb.	Live load + impact moment ft.lb.	Total moment ft.lb.
1	$.00L_1$	0	0	0
	$.38L_1$	14,100	19,000	33,100
	$.40L_1$	13,900	19,100	33,000
	$.70L_1$	1,500	14,500	16,000
	$.70L_1$	1,500	−7,900	−6,400
	$1.00L_1$	−30,500	−14,800	−45,300
2	$.00L_1$	−30,500	−14,800	−45,300
	$.20L_2$	0	11,100	11,100
	$.20L_2$	0	−6,600	−6,600
	$.50L_2$	18,000	19,300	37,300
	$.80L_2$	2,700	10,400	13,100
	$.80L_2$	2,700	−4,600	−1,900
	$1.00L_2$	−26,000	−13,900	−39,900
3	$.00L_3$	−26,000	−13,900	−39,900
	$.30L_3$	−2,900	13,000	10,100
	$.30L_3$	−2,900	−8,800	−11,700
	$.60L_3$	7,100	15,900	23,000
	$.63L_3$	7,400	15,800	23,200
	$1.00L_3$	0	0	0

so only the final moments are shown in the table above.

Having the moments at critical sections and at intermediate points, the positive moment curves can be drawn by passing a parabola through the plotted points indicated thus, ⊙, in Fig. 34 and straight lines through the

Fig. 34



points of negative moment similarly indicated. From Fig. 34 it will be seen that this method of drawing the moment curves is sufficiently accurate since moments computed at points other than those used in the problem as indicated thus, •, fall on the parabolas in the case of the positive moments, and in the case of the negative moments, though not falling exactly on the straight lines, the computed values are always less than corresponding values on the straight lines within the limits where the negative moments are important.

By means of Fig. 34 the area of reinforcement required at any section can be determined and the location of cut-offs established. Fig. 44 shows arrangement of reinforcement for Design Problem No. 2, which is essentially the same as far as the deck is concerned as that which would be used for this problem except that the quantity is different.

Step 7. Shears

Check shear at critical sections. In this case maximum shear is obviously just to the right of support *B* when the large wheel of the leading truck is at that point as shown in Fig. 35.

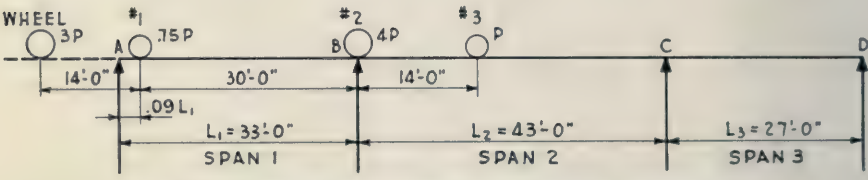


Fig. 35. Loading for maximum shear at right of support *B*.

$$\text{Dead load shear} = \frac{200 \times 43}{2} + \frac{(152.7 - 130.1)200}{43} = 4{,}410 \text{ lb.}$$

$$\text{Live load shear} = \frac{(0.021 \times 0.75P \times 33 + 0.086P \times 43)}{43}$$

$$- \frac{(-0.007 \times 0.75P \times 33 + 0.063P \times 43)}{43} + 4.675P$$

$$= 4.7P = 4.7 \times 625 = \underline{2{,}940 \text{ lb.}}$$

$$\text{Total shear} = 7{,}350 \text{ lb.}$$

$$\text{Unit shear } V = \frac{7{,}350}{12 \times 0.9 \times 14} = 49 \text{ p.s.i.}$$

The unit shear is within the allowable for the 3,000-lb. concrete assumed for the design so it will not be necessary to haunch the slab.

DESIGN PROBLEM No. 2

In order to show the effect of making the deck integral with the interior piers as compared with a freely supported deck, the three-span bridge in Design Problem No. 1 will be redesigned assuming the deck and piers to be integral at B and C (*see Fig. 36*), otherwise all conditions are assumed to be identical. Detailed computations will not be given except as they are necessary to illustrate the effect of the restraint at the supports on the design calculations or procedure. The results obtained for the freely supported and the restrained condition are tabulated in several instances for ready comparison.

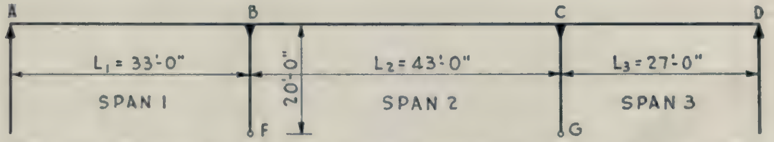


Fig. 36. Design Problem No. 2, deck integral with piers at B and C.

Step 1—Select r Values

For the uniform deck depth all r values will be zero as in Problem No. 1.

Step 2—Draw Influence Lines for Moment at Supports

For simplicity of the demonstration, piers BF and CG will be assumed of uniform cross section and of the same thickness as the deck slab. By so doing the values of I_c in the equation for D_{BA} below will cancel; otherwise it would be necessary to use the moment of inertia at the section of reference for each member involved*. As in Problem No. 1, all stiffness coefficients, k, equal 4.0 and all carry-over factors, C, equal -0.5.

Assuming the piers hinged at F and G, the stiffness coefficients k_{BF} and k_{CG} must be modified when determining the distribution factors as was done in the case of deck members BA and CD.

Therefore:

$$D_{BA} = \frac{\dfrac{0.75 \times 4I_c}{33}}{\dfrac{0.75 \times 4I_c}{33} + \dfrac{4I_c}{43} + \dfrac{0.75 \times 4I_c}{20}} = 0.272$$

Similarly, all other distribution factors can be obtained.

Table VII—Distribution Factors

	D_{BA}	D_{BC}	D_{BF}	D_{CB}	D_{CD}	D_{CG}
Supported deck	0.494	0.506	.....	0.456	0.544	.....
Integral deck	0.272	0.279	0.449	0.263	0.314	0.423

*See second footnote page 15.

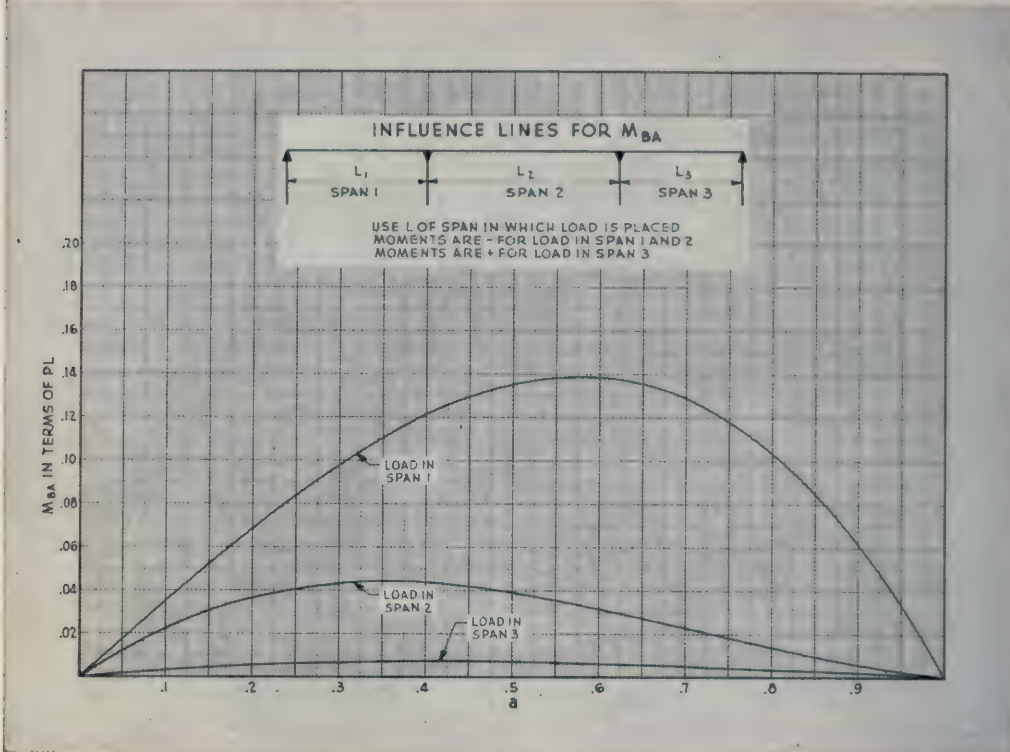

INFLUENCE LINES FOR M_{BA}
L_1
SPAN 1
L_2
SPAN 2
L_3
SPAN 3
USE L OF SPAN IN WHICH LOAD IS PLACED
MOMENTS ARE - FOR LOAD IN SPAN 1 AND 2
MOMENTS ARE + FOR LOAD IN SPAN 3
M_{BA} IN TERMS OF PL
.20
.18
.16
.14
.12
.10
.08
.06
.04
.02
LOAD IN SPAN 1
LOAD IN SPAN 2
LOAD IN SPAN 3
.1
.2
.3
.4
.5
.6
.7
.8
.9
a

Fig. 37

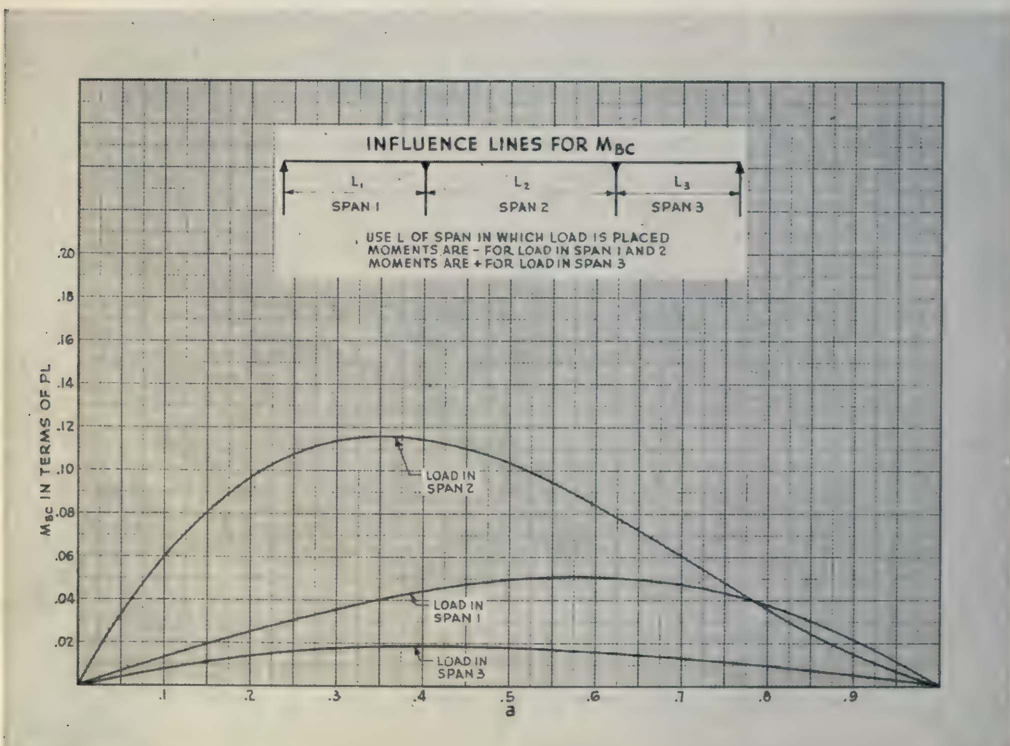

INFLUENCE LINES FOR M_{BC}
L_1
SPAN 1
L_2
SPAN 2
L_3
SPAN 3
USE L OF SPAN IN WHICH LOAD IS PLACED
MOMENTS ARE - FOR LOAD IN SPAN 1 AND 2
MOMENTS ARE + FOR LOAD IN SPAN 3
M_{BC} IN TERMS OF PL
.20
.18
.16
.14
.12
.10
.08
.06
.04
.02
LOAD IN SPAN 2
LOAD IN SPAN 1
LOAD IN SPAN 3
.1
.2
.3
.4
.5
.6
.7
.8
.9
a

Fig. 38

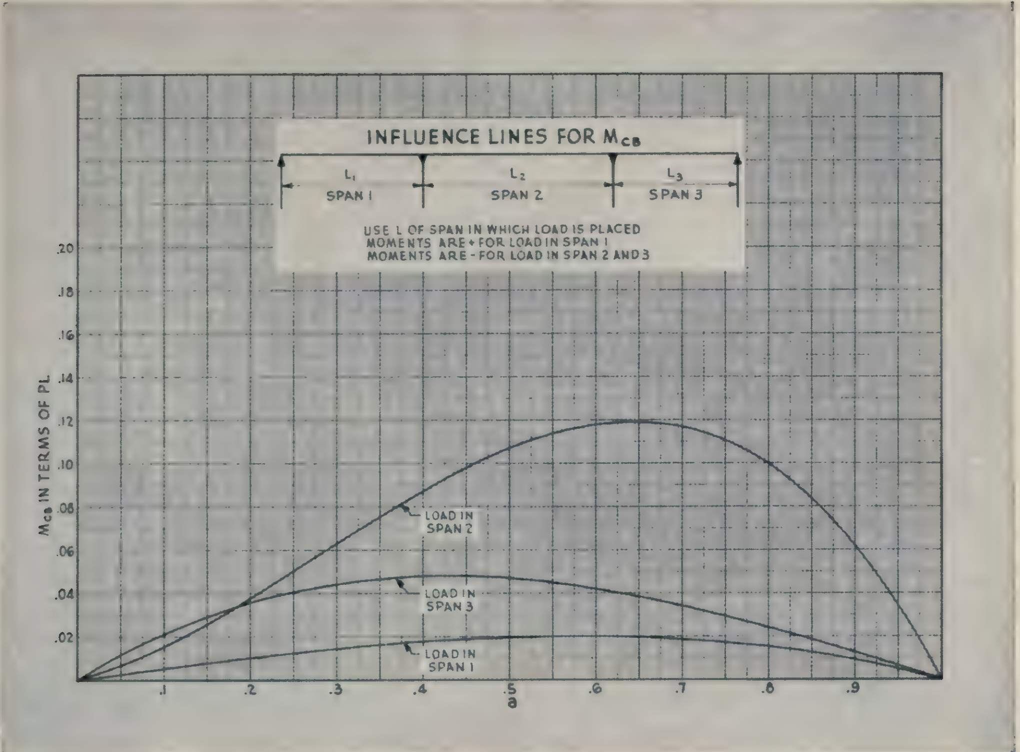

INFLUENCE LINES FOR M_{CB}
L_1
SPAN 1
L_2
SPAN 2
L_3
SPAN 3
USE L OF SPAN IN WHICH LOAD IS PLACED
MOMENTS ARE + FOR LOAD IN SPAN 1
MOMENTS ARE - FOR LOAD IN SPAN 2 AND 3
M_{CB} IN TERMS OF PL
LOAD IN SPAN 2
LOAD IN SPAN 3
LOAD IN SPAN 1
a

Fig. 39

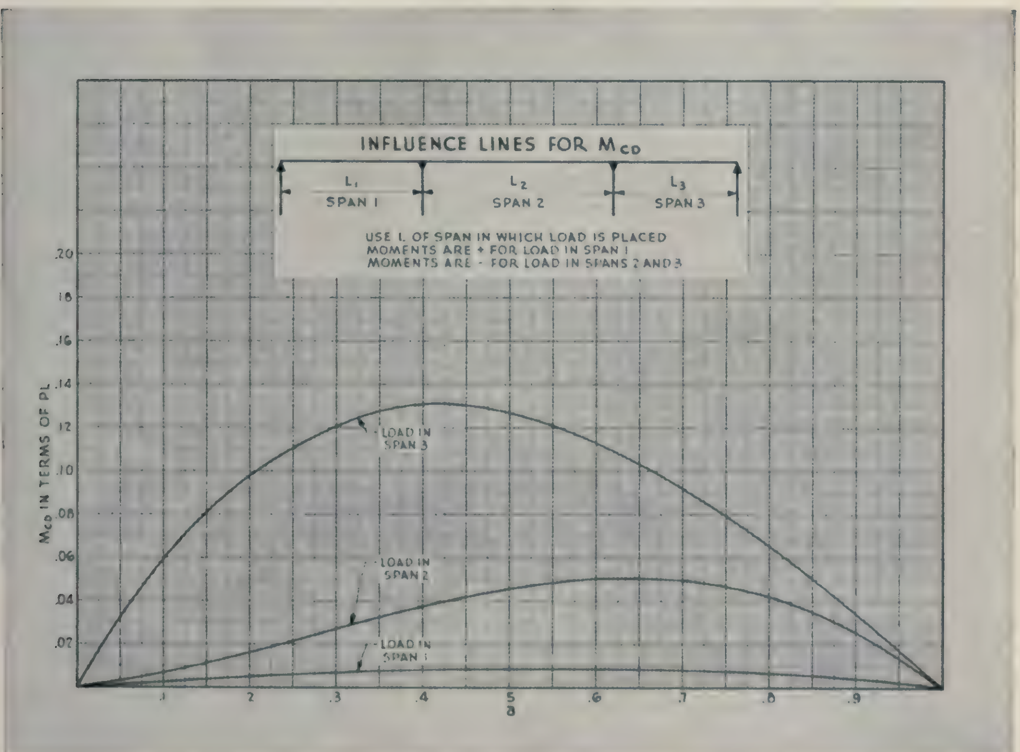

INFLUENCE LINES FOR M_{CD}
L_1
SPAN 1
L_2
SPAN 2
L_3
SPAN 3
USE L OF SPAN IN WHICH LOAD IS PLACED
MOMENTS ARE + FOR LOAD IN SPAN 1
MOMENTS ARE - FOR LOAD IN SPANS 2 AND 3
M_{CD} IN TERMS OF PL
LOAD IN SPAN 3
LOAD IN SPAN 2
LOAD IN SPAN 1
a

Fig. 40

Having the distribution factors, the moments at the supports are computed by the formulas in Table I as a load P moves across the bridge just as in the preceding problems.

Ordinates for the influence lines shown in Figs. 37 to 40 can now be obtained simply by substitution of the fixed end moments due to the load P at successive tenth points of each span in the expressions for moment for the integral deck in Table VIII.

TABLE VIII—Moments in Deck at Supports

	M_{BA}	M_{BC}	M_{CB}	M_{CD}
Load in Span 1				
Supported deck	$0.476M_1$	$0.476M_1$	$-0.146M_1$	$-0.146M_1$
Integral deck	$0.723M_1$	$0.266M_1$	$-0.105M_1$	$-0.045M_1$
Load in Span 2				
Supported deck	$0.524M^F_{BC} + 0.120M^F_{CB}$	$0.524M^F_{BC} + 0.120M^F_{CB}$	$0.146M^F_{BC} + 0.578M^F_{CB}$	$0.146M^F_{BC} + 0.578M^F_{CB}$
Integral deck	$0.277M^F_{BC} + 0.037M^F_{CB}$	$0.734M^F_{BC} + 0.097M^F_{CB}$	$0.105M^F_{BC} + 0.751M^F_{CB}$	$0.045M^F_{BC} + 0.320M^F_{CB}$
Load in Span 3				
Supported deck	$-0.120M_3$	$-0.120M_3$	$0.423M_3$	$0.423M_3$
Integral deck	$-0.037M_3$	$-0.097M_3$	$0.249M_3$	$0.680M_3$

Step 3. Maximum Live Load Moments at Critical Sections

Where the deck is integral with the piers, as in this problem, different positions of loading from those used when the deck was freely supported will be necessary to give maximum moment at each side of the piers; otherwise the procedure for obtaining moments by means of the influence lines is identical with that given in detail in Problem No. 1. The restraint at B and C will tend to shift the section of maximum positive moment in the end spans slightly closer to the exterior supports. Table IX shows comparative values for the integral and freely supported deck.

TABLE IX—Maximum Live Load Moments at Critical Sections—Ft.Lb.

	$0.35L_1$	$0.38L_1$	$0.40L_1$	$1.00L_1$ (M_{BA})	$0.00L_2$ (M_{BC})	$0.50L_2$	$1.00L_2$ (M_{CB})	$0.00L_3$ (M_{CD})	$0.58L_3$	$0.60L_3$	$0.63L_3$
Supported deck	18,700	19,000	19,100	– 14,800	– 14,800	19,300	– 13,900	– 13,900	16,000	15,900	15,800
Integral deck	16,800	17,000	16,900	– 15,400	– 16,100	16,900	– 15,400	– 11,900	13,900	13,900	13,800

Step 4. Dead Load Moments

The dead load moments at the supports in terms of w can be determined simply by substitution of the fixed end moment due to w in the expressions for moment given in Table VIII just as in Step 4 of the preceding problem. Comparative results are shown in Table X.

Table X—Dead Load Moments at Supports

	M_{BA}	M_{BC}	M_{BF}	M_{CB}	M_{CD}	M_{CG}
Supported deck	$-152.7w$	$-152.7w$	.....	$-130.1w$	$-130.1w$	.....
Integral deck	$-143.3w$	$-155.4w$	$-12.1w$	$-140.2w$	$-112.1w$	$-28.1w$

Step 5. Assume Depth at Center of Span

It is obvious that M_{BC} will determine the depth of slab required. Assuming $h_c = 16$:

$M_{BC} = -155.4 \times 200 - 16{,}100 = -47{,}200$ ft.lb. (*see Tables IX and X*)

$$\text{Required } h_c = 2 + \sqrt{\frac{47{,}200}{248}} = 15.8, \text{ say 16 in.}$$

Step 6. Draw Maximum Moment Curves

In order to draw maximum moment curves it will be necessary to compute dead load moments and maximum live load moments at intermediate points between supports. The procedure is the same as in Problem No. 1 and therefore has not been repeated. The moments at intermediate points as well as those at critical sections are given in Table XI, and Fig. 41 shows the curves which have been drawn as in Problem No. 1 by passing a para-

Table XI—Maximum Moments at Sections Required for Drawing Moment Curves

Span	Section	Dead load moment ft.lb.	Live load + impact moment ft.lb.	Total moment ft.lb.
1	$.00L_1$	0	0	0
	$.38L_1$	14,800	17,000	31,800
	$.40L_1$	14,700	16,900	31,600
	$.70L_1$	2,800	10,800	13,600
	$.70L_1$	2,800	−3,900	−1,100
	$1.00L_1$	−28,700	−15,500	−44,200
2	$.00L_2$	−31,100	−16,100	−47,200
	$.20L_2$	−900	8,200	7,300
	$.20L_2$	−900	−3,600	−4,500
	$.50L_2$	16,700	16,900	33,600
	$.80L_2$	1,000	8,200	9,200
	$.80L_2$	1,000	−2,600	−1,600
	$1.00L_2$	−28,000	−15,400	−43,400
3	$.00L_3$	−22,400	−11,900	−34,300
	$.30L_3$	−400	9,400	9,000
	$.30L_3$	−400	−4,600	−5,000
	$.60L_3$	8,500	13,900	22,400
	$1.00L_3$	0	0	0

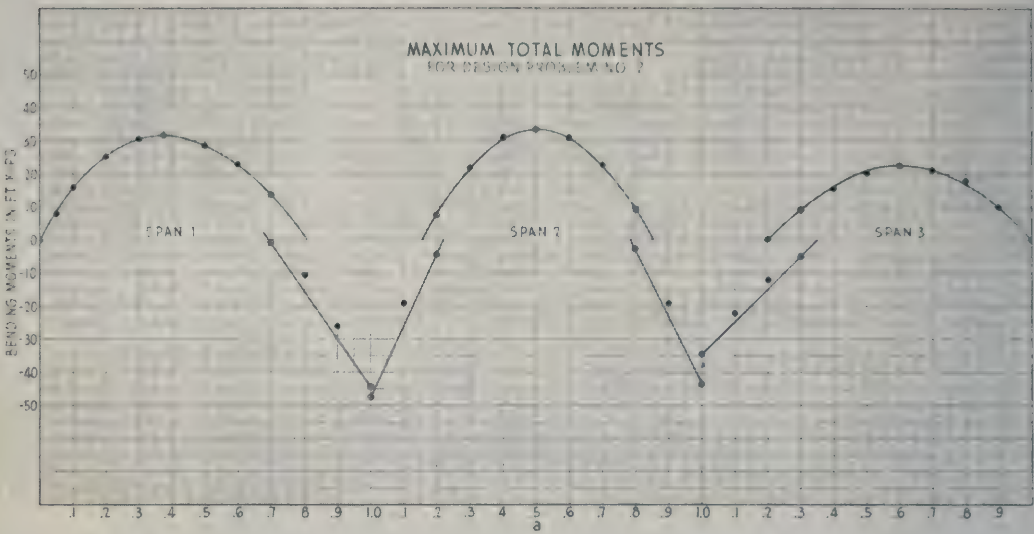


Fig. 41

bola through the plotted points for positive moment and straight lines through the points of negative moment.

Step 7. Shears

Maximum shear occurs just to the left of support C with the loads as hown in Fig. 42.

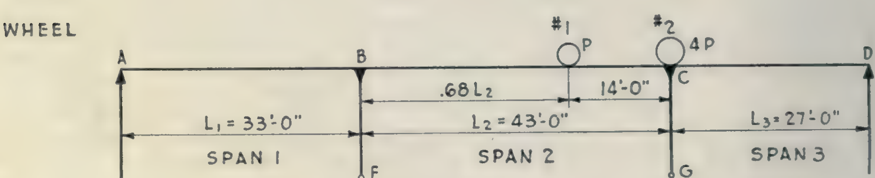


Fig. 42. Loading for maximum shear at left of support *C*.

$$\text{Dead load shear} = 200 \times 21.5 - \frac{31{,}100 - 28{,}000}{43} = 4{,}230 \text{ lb.}$$

$$\text{Live load shear} = 4.68P + \left(\frac{5.11 - 2.86}{43}\right)P$$

$$= 4.73P = 4.73 \times 649 \qquad = \underline{3{,}070} \text{ lb.}$$

$$\text{Total shear} = 7{,}300 \text{ lb.}$$

Step 8. Maximum Moment in Piers

Maximum pier moment will occur in pier C when span BC is loaded as shown in Fig. 43 and span CD is not loaded.

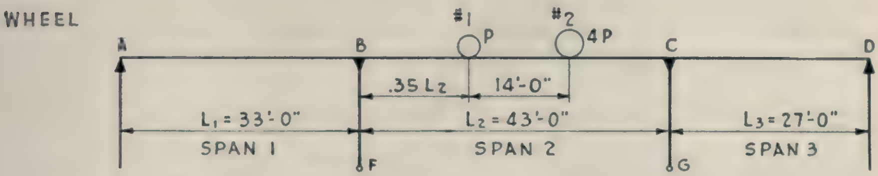


Fig. 43. Loading for maximum moment in pier *CG*.

For live load:

$$M_{CB} = -0.075P \times 43 - 0.119 \times 4P \times 43 = -23.7P$$

$$M_{CD} = -0.032P \times 43 - 0.050 \times 4P \times 43 = \underline{-10.0P}$$

$$M_{CB} - M_{CD} = -13.7P$$

$$= -13.7 \times 649 = -8{,}900 \text{ ft.lb.}$$

Dead load:

$$M_{CB} - M_{CD} = (-140.2 + 112.1)\ 200 \qquad = \underline{-\ 5{,}600 \text{ ft.lb.}}$$

$$M_{CG} = -14{,}500 \text{ ft.lb.}$$

Step 9. Moments Due to Change in Deck Length

Change in deck length due to -45 deg. change in temperature
$= 43 \times 12 \times 45 \times 0.000006 = 0.139$ in. shortening.

Change in deck length due to shrinkage
$= 43 \times 12 \times 0.0002 = 0.103$ in. shortening.

Deflection of each pier is one-half of the total change in deck length between the piers because piers are identical, or

$$= 0.5(0.139 + 0.103) = 0.121 \text{ in.}$$

Since the piers are solid and of uniform section, equal in height, and are assumed to be hinged at the bottom, the resultant moment due to the deflection can be obtained from Equation 15 as:

$$M^F_{BF} = M^F_{CG} = \frac{3 \times 3 \times 10^6 \times 16^3 \times 0.121}{240^2 \times 12} = 6{,}500 \text{ ft.lb.}$$

It should be noted that M^F_{BF} and M^F_{CG} are positive according to the sign convention because they cause compression on the outside of the piers with reference to the centerline of the bridge.

The moments in the deck at supports and at the tops of the piers can be computed by substitution of the fixed end moments and the physical constants of the members in the formulas in Table II. The algebraic sum of the moments due to the displacement of the left and right piers gives the desired final moment.

$$M_{BA} = -\frac{D_{BA}}{1-U} M^F_{BF} + \frac{C_{CB}D_{CB}D_{BA}}{1-U} M^F_{CG}$$

$$= -\frac{0.272}{1 - 0.5 \times 0.5 \times 0.279 \times 0.263} \times 6{,}500$$

$$+ \frac{-0.5 \times 0.263 \times 0.272}{1 - 0.018} \times 6{,}500 = -2{,}040 \text{ ft.lb.}$$

$$M_{BF} = \frac{1 - D_{BF} - U}{1-U} M^F_{BF} + \frac{C_{CB}D_{CB}D_{BF}}{1-U} M^F_{CG}$$

$$= \frac{1 - 0.449 - 0.018}{0.982} \times 6{,}500$$

$$+ \frac{-0.5 \times 0.263 \times 0.449}{0.982} \times 6{,}500 = 3{,}130 \text{ ft.lb.}$$

$$M_{BC} = \frac{D_{BC} - U}{1-U} M^F_{BF} + \frac{C_{CB}D_{CB}(1 - D_{BC})}{1-U} M^F_{CG}$$

$$= \frac{0.279 - 0.018}{0.982} \times 6{,}500$$

$$+ \frac{-0.5 \times 0.263(1 - 0.279)}{0.982} \times 6{,}500 = 1{,}100 \text{ ft.lb.}$$

$$M_{FB} = 0$$

$$M_{GC} = 0$$

$$M_{CB} = \frac{C_{BC}D_{BC}(1 - D_{CB})}{1-U} M^F_{BF} + \frac{D_{CB} - U}{1-U} M^F_{CG}$$

$$= \frac{-0.5 \times 0.279(1 - 0.263)}{0.982} \times 6{,}500$$

$$+ \frac{0.263 - 0.018}{0.982} \times 6{,}500 = 940 \text{ ft.lb.}$$

$$M_{CG} = \frac{C_{BC}D_{BC}D_{CG}}{1-U} M^F_{BF} + \frac{1 - D_{CG} - U}{1-U} M^F_{CG}$$

$$= \frac{-0.5 \times 0.279 \times 0.423}{0.982} \times 6{,}500$$

$$+ \frac{1 - 0.423 - 0.018}{0.982} \times 6{,}500 = 3{,}310 \text{ ft.lb.}$$

$$M_{CD} = \frac{C_{BC}D_{BC}D_{CD}}{1-U} M^F_{BF} - \frac{D_{CD}}{1-U} M^F_{CG}$$

$$= \frac{-0.5 \times 0.279 \times 0.314}{0.982} \times 6{,}500 - \frac{0.314}{0.982} \times 6{,}500 = -2{,}370 \text{ ft.lb.}$$

According to the adopted sign convention it will be seen that the result of deck shortening is to cause tension in the top of the deck at left of pier BF and right of pier CG because M_{BA} and M_{CD} are negative, and because

M_{BC} and M_{CB} are positive there will be compression in the top of the deck at their respective sections. Likewise there will be compression on the outside of the piers since M_{BF} and M_{CG} are positive. Deck lengthening would have just the opposite effect at all sections. In this problem the increase in moments at the critical sections due to changes in deck length is less than the increase of 25 per cent usually allowed in the working stress for combined moment, so the moments due to deck load only will govern in the design. For longer spans, however, the combined moments may be critical.

Step 10. Selection and Arrangement of Reinforcement

Fig. 44, which is a longitudinal section through the deck, shows an arrangement of longitudinal and transverse reinforcement. The area of longitudinal bars and the locations of cut-offs are in accordance with the requirements indicated by the curve of maximum moments, Fig. 41. For simplicity in placing, straight bars only have been used in the deck.

The transverse reinforcement is 25 per cent of the longitudinal steel which is in accordance with the recommendation in the Bureau of Public Roads bulletin referred to in third footnote on page 10.

Bars from the piers sufficient to resist the moment in the tops of the piers extend into the deck far enough to develop their strength in bond. Half of the bars may be stopped off at mid-heights of the pier since the moment varies from maximum at the top to zero at the bottom, the pier having been assumed hinged at the base. In general the same amount of reinforcement will be used in each pier and it will be arranged symmetrically about the center of the pier.

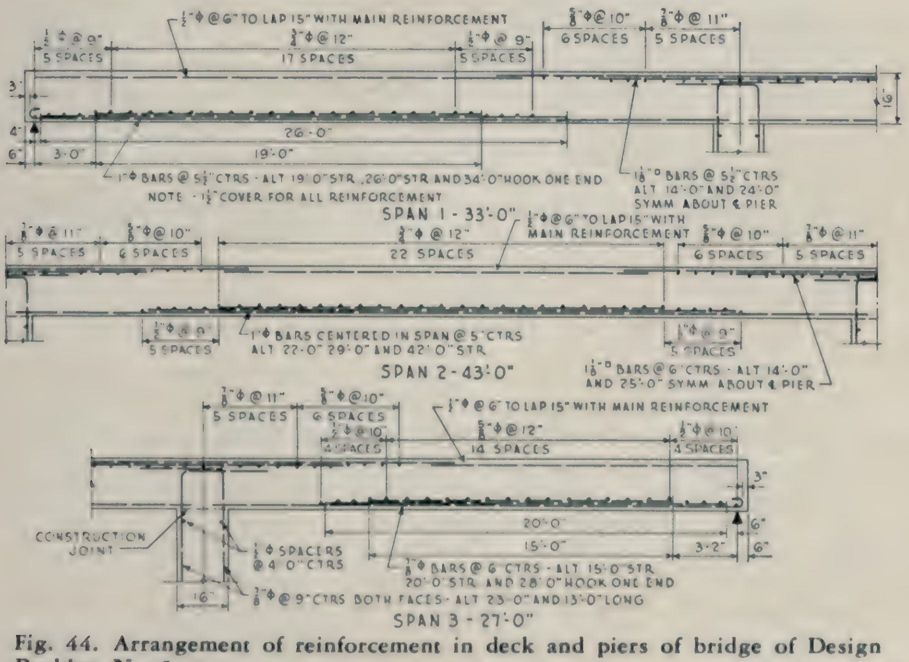


Fig. 44. Arrangement of reinforcement in deck and piers of bridge of Design Problem No. 2.

Section VI—Design Procedure for T-Girder Bridges

When studying the design procedure for T-girder bridges presented here, it is suggested that reference be made to Design Problem No. 3. To do so will aid in becoming familiar with each step of the procedure.

1. Select r *Values*

As in the case of slab bridges, r values can be chosen at the outset which will result in a satisfactory design and which will automatically determine the depth of girders for definite girder spacing and width of stem.

For all continuous girder units having span ratios of 1:1.37 . . . :1 as recommended in Section II—Layout: r for outer ends of end spans may be zero, or if haunched, 0 to 1.3 as desired; for a three-span unit, r at each side of the two intermediate supports may be taken as 1.3; for a four-span unit r for each side of center support may be 1.5 and at the first interior supports may be assumed as 1.3. It may be necessary to modify these values for designs based on dead load to live load ratios or span length ratios greatly different from those used in this booklet.

2. Draw Influence Lines

The procedure outlined under Step 2 for slab bridges applies to girder bridges as well.

3. Maximum Live Load Moments at Critical Sections

Determination of live load moments is essentially the same as outlined for slab bridges in Step 3.

4. Dead Load Moments

As for slab bridges, Step 4, dead load moments can also be found for girder bridges in terms of the yet unknown uniform dead load w lb. per lin. ft. and W_{BA}, W_{BC}, etc., the haunch load.

5. Select Girder Spacing, Determine Girder Width and Design Slab

Spacing of girders will depend upon over-all width of deck and to some extent upon available headroom. Economical spacing depends largely on slab thickness and it will usually be found most economical to select a spacing between 7 and 10 ft.

Having decided on the spacing of girders, choose the width of girder stem, b'. In reality, width of stem is dependent upon girder spacing, slab thickness, and length of span, but usually it is even more dependent upon the arrangement of reinforcement than upon moment and shear stresses. A fair approximation can be had in terms of two variables, such as $b' = 0.0025 \sqrt{b} \times L$, where b = spacing center to center of girders and L is the length

of the end span. For example: Spacing $b = 8$ ft. 6 in. and end span of a unit is 55 ft. 0 in.; then required $b' = 0.0025\sqrt{102} \times 660 = 16.7$ in. A width of 16½ in. will provide for four 1¼-in. square bars in each layer of reinforcement at points of critical positive moments. It is often desirable to place a considerable area of steel in each layer so as to avoid an arrangement of bars that would place reinforcement too near the neutral axis.

Now determine slab thickness required. The bending moment in 6 and 7-in. slabs due to wheel load concentrations may be taken from Fig. 45*. The assumed thickness can then be checked to see if allowable stress requirements are satisfied.

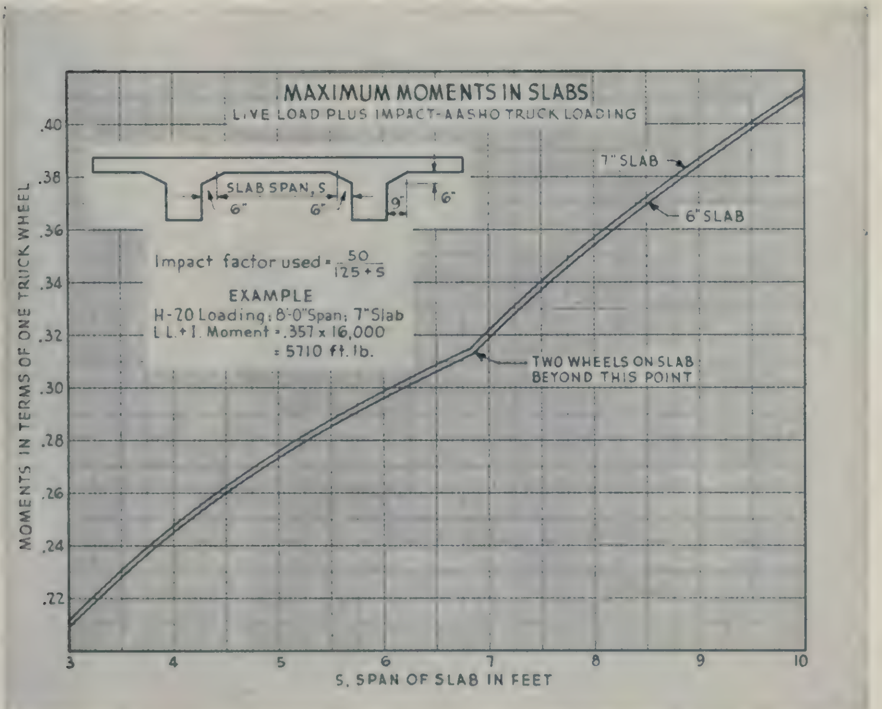


Fig. 45

6. *Assume Trial Depth at Supports*

Having selected the girder spacing, determined the width of stem and found the required slab thickness, compute and plot a moment of inertia curve for the gross T-section center to center of girders. Since the only variable in girder section is the depth, h, the moment of inertia curve, should be plotted for values of h between $0.04L$ and $0.12L$; L being the length of the end span. Coefficients for moment of inertia of T-sections may be taken from Fig. 46.

*"Computation of Stresses in Bridge Slabs Due to Wheel Loads" by H. M. Westergaard, *Public Roads*, March 1930, pages 1—23.

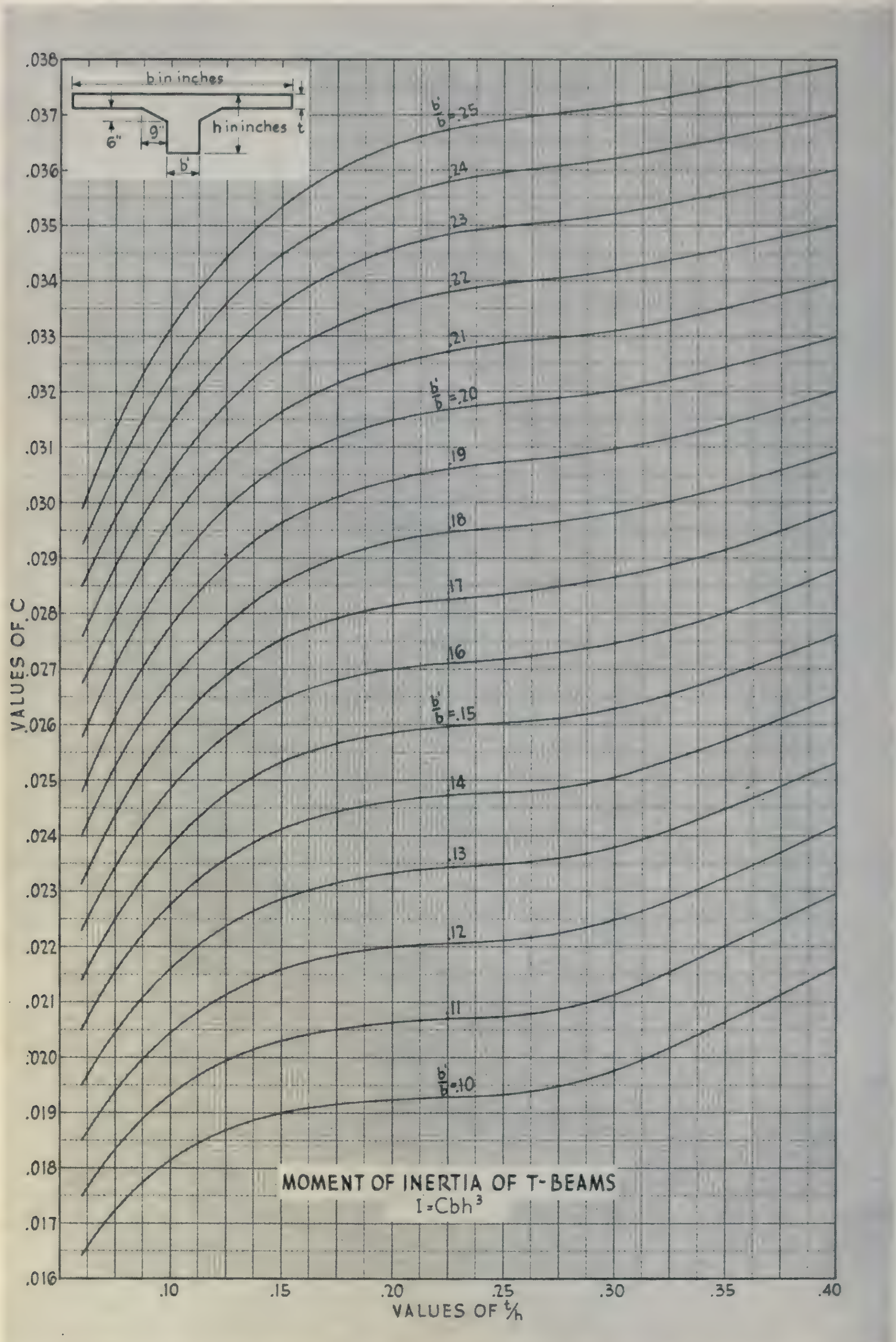

b in inches
h in inches
t
6"
9"
b'
$\frac{b'}{b}$=.25
.24
.23
.22
.21
$\frac{b'}{b}$=.20
.19
.18
.17
.16
$\frac{b'}{b}$=.15
.14
.13
.12
.11
$\frac{b'}{b}$=.10
VALUES OF C
.038
.037
.036
.035
.034
.033
.032
.031
.030
.029
.028
.027
.026
.025
.024
.023
.022
.021
.020
.019
.018
.017
.016
.10
.15
.20
.25
.30
.35
.40
VALUES OF t/h
MOMENT OF INERTIA OF T-BEAMS
I = Cbh³

Fig. 46

Since depths at supports are dependent upon spacing of girders, width of stem and span lengths, they can be expressed as a function of these three variables. For the conditions of loading and allowable stresses used in this booklet the following formula will give good tentative depths:

$$h_s = 1.93(b')^{-0.11}b^{0.25}L^{0.83}$$

where b' = width of stem in inches, b = spacing center to center of girder in feet and L = length of end span of continuous unit in feet, h_s, depth at support, will be in inches. Solution of the foregoing exponential equation is not necessary as values of h_s can be obtained directly from Fig. 47.

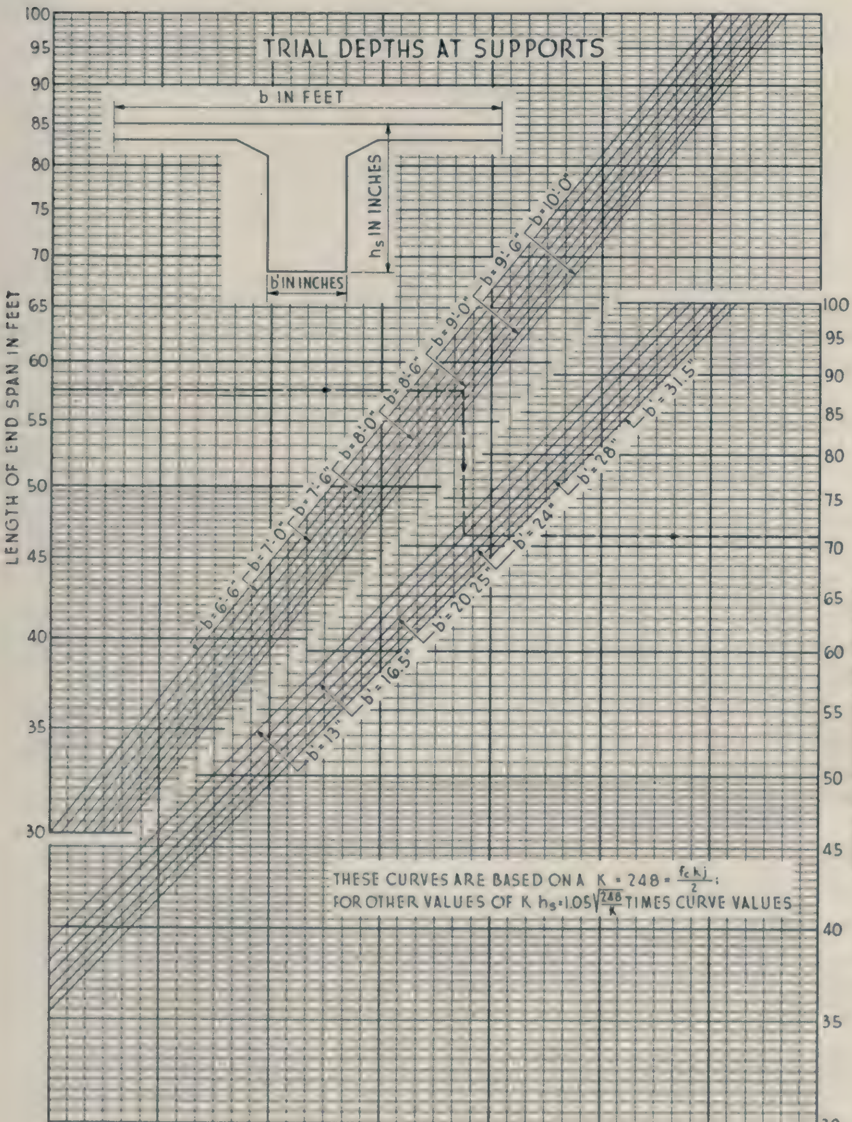


Fig. 47

After obtaining h_s enter the moment of inertia curve just plotted for the girders being designed and find the corresponding value of I_s (moment of inertia at support) and divide it by $(1 + r)^3$; then go back into the curve with this value to find the corresponding h which will be the depth at the center h_c. Now w, W_{BA}, W_{BC}, etc., can be computed.

7. *Check Assumed Sections*

Substitute the values of w, W_{BA}, W_{BC}, etc., in the dead load moments found under Step 4; add them to the live load moments found under Step 3 and check required sections against those assumed. If too large or too small, assume new h_s and repeat procedure given in last paragraph in Step 6.

8. *Curves of Maximum Moments*

After satisfactory h_s and h_c values have been found, complete moment curves as described in Step 6 under slab bridges.

9. *Maximum Total Shears*

Find maximum total shears on each side of supports and at 0.4 and 0.6 points in each span. Shears at 0.0 and 0.4 points will be of one sign; those at 0.6 and 1.0 points are opposite in sign to those at 0.0 and 0.4 points. Draw straight lines between 0.0 and 0.4, and between 0.6 and 1.0 points until they intersect. These lines give maximum shears accurately enough for design purposes.

10. *Moments in Piers Due to Deck Loads*

and

11. *Moments Due to Changes in Deck Length*

The procedure for determining moments in piers due to deck loads and moments throughout the structure due to changes in deck length when the deck and piers are integral is the same as for slab bridges. Because the length of deck between the first and third interior supports in a four-span bridge is usually greater than the interior span of a three-span structure, the moments caused by changes in deck length are more likely to result in critical moments when combined with those due to dead and live load.

DESIGN PROBLEM No. 3

Fig. 48 shows a four-span continuous girder bridge, laid out in accordance with principles discussed in Section II—Layout. Assume the deck to be freely supported and the design to be made for A.A.S.H.O. H-20 truck-train loading plus impact; 10-ft. traffic lanes; f_c = 1,200 p.s.i. for positive moments and 1,350 p.s.i. for negative moments; f_s = 18,000 p.s.i.

Step 1. Select r *Values*

$r_{AB} = 0; r_{BA} = r_{BC} = 1.3; r_{CB} = r_{CD} = 1.5$
h_c will be assumed the same for all spans.

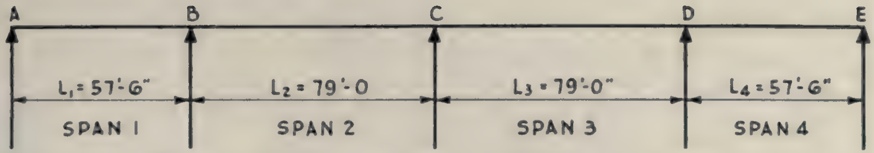


Fig. 48. Design Problem No. 3, deck freely supported.

Step 2. Draw Influence Lines

With the above r values obtain stiffness coefficients from Fig. 6 as.

$$k_{BA} = 10.47; k_{BC} = 15.54; k_{CB} = 16.47$$

Next with the r values enter Fig. 5 and obtain carry-over factors:

$$C_{AB} = -0.888; C_{BA} = -0.417; C_{BC} = -0.758; \text{and } C_{CB} = -0.717$$

With these carry-over and stiffness factors, distribution factors are found by the use of Equations 10 and 11:

$$D_{BA} = \frac{(1 - 0.888 \times 0.417)\ 10.47^*}{(1 - 0.888 \times 0.417)\ 10.47 + \frac{15.54}{1.37}} = 0.368$$

and since the deck is not integral with piers,

$$D_{BC} = 1.000 - 0.368 = 0.632$$

By symmetry

$$D_{CB} = 0.500$$

Next determine final moments at supports B, C and D in terms of fixed end moments by use of formulas in Table I.

Load in Span 1

Since the deck is freely supported at B:

$$M_{BA} = M_{BC} = \frac{(1 - D_{BA}) - (2 - D_{BA})U}{1 - 2U} M_1 \qquad U = C_{BC}C_{CB}D_{BC}D_{CB}$$

$$U = -0.758 \times (-0.717) \times 0.632 \times 0.5 = 0.172$$

$$M_{BA} = M_{BC} = \frac{(1 - 0.368) - (2 - 0.368)\ 0.172}{1 - 2 \times 0.172} M_1 = 0.536\ M_1 \qquad (44)$$

and likewise

$$M_{CB} = M_{CD} = \frac{C_{BC}D_{BC}\ (1 - D_{CB} - U)}{1 - 2U} M_1$$

$$= \frac{-0.758 \times 0.632\ (1 - 0.5 - 0.172)}{0.656} M_1 = -0.240\ M_1 \ (45)$$

also

$$M_{DC} = M_{DE} = \frac{U(1 - D_{BC})}{1 - 2U} M_1 = \frac{0.172\ (1 - 0.632)}{0.656} M_1 = 0.096\ M_1 \quad (46)$$

*Since h_c is the same for all spans, I_c is the same for all spans and the numerator and denominator can be divided through by I_c/L_1.

Load in Span 2

$$M_{BA} = M_{BC} = \frac{D_{BA}(1-U)M^F_{BC} - WM^F_{CB}}{1-2U} \qquad W = C_{CB}D_{CB}D_{BA}$$

$$W = -0.717 \times 0.5 \times 0.368 = -0.132$$

$$M_{BA} = M_{BC} = \frac{0.368\,(1-0.172)\,M^F_{BC} + 0.132\,M^F_{CB}}{0.656}$$

$$= 0.465M^F_{BC} + 0.201M^F_{CB} \tag{47}$$

$$M_{CB} = M_{CD} = \frac{-C_{BC}D_{BC}(1-D_{CB}-U)M^F_{BC} + (1-D_{CB}-U)M^F_{CB}}{1-2U}$$

$$= \frac{0.758 \times 0.632\,(1-0.5-0.172)M^F_{BC} + 0.328M^F_{CB}}{0.656}$$

$$= 0.240M^F_{BC} + 0.500M^F_{CB} \tag{48}$$

$$M_{DC} = M_{DE} = \frac{-UD_{BA}M^F_{BC} + WM^F_{CB}}{1-2U}$$

$$= \frac{-0.172 \times 0.368M^F_{BC} - 0.132M^F_{CB}}{0.656}$$

$$= -0.096M^F_{BC} - 0.201M^F_{CB} \tag{49}$$

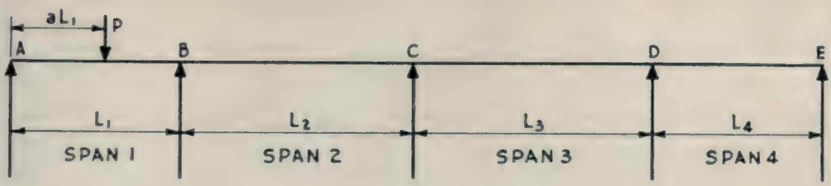
Due to symmetry, moments for loads in Span 3 and Span 4 may be obtained by interchanging subscripts corresponding to each other about the centerline of unit.

Ordinates for influence lines for final distributed moments at supports for a moving concentrated load can be obtained now by substituting the fixed end moments taken from Figs. 7 to 15 in Equations 44 to 49. For convenience a load P has been placed at each tenth point consecutively and tabulated in Tables XII and XIII opposite.

Open abutments afford clear vision at grade separation in Belknap County, N. H., over Boston and Maine Railroad. Designed by New Hampshire State Highway Department, John W. Childs, bridge engineer.

Table XII—Ordinates for Influence Lines for Moments at Supports

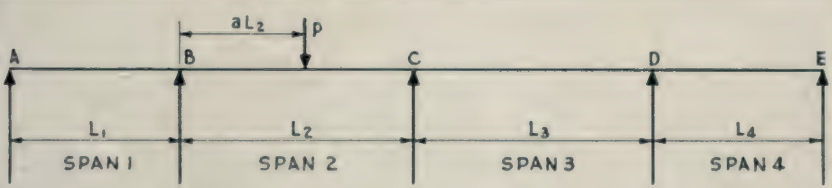
Load in Span 1



a	M^F_{AB} (Figs. 7-15)	M^F_{BA} (Figs. 7-15)	M_1 (Eq. 1, pg. 17)	$M_{BA}=M_{BC}$ (Eq. 44)	$M_{CB}=M_{CD}$ (Eq. 45)	$M_{DC}=M_{DE}$ (Eq. 46)
.1	$-.077\ PL_1$	$-.019\ PL_1$	$-.087\ PL_1$	$-.047\ PL_1$	$+.021\ PL_1$	$-.008\ PL_1$
.2	$-.114$ "	$-.068$ "	$-.169$ "	$-.091$ "	$+.041$ "	$-.016$ "
.3	$-.121$ "	$-.130$ "	$-.238$ "	$-.128$ "	$+.057$ "	$-.023$ "
.4	$-.106$ "	$-.191$ "	$-.285$ "	$-.153$ "	$+.068$ "	$-.027$ "
.5	$-.079$ "	$-.237$ "	$-.307$ "	$-.165$ "	$+.074$ "	$-.030$ "
.6	$-.050$ "	$-.253$ "	$-.297$ "	$-.159$ "	$+.071$ "	$-.029$ "
.7	$-.026$ "	$-.231$ "	$-.254$ "	$-.136$ "	$+.061$ "	$-.024$ "
.8	$-.010$ "	$-.175$ "	$-.184$ "	$-.098$ "	$+.044$ "	$-.018$ "
.9	$-.002$ "	$-.095$ "	$-.097$ "	$-.052$ "	$+.023$ "	$-.009$ "

Table XIII—Ordinates for Influence Lines for Moments at Supports

Load in Span 2



a	M^F_{BC} (Figs. 7-15)	M^F_{CB} (Figs. 7-15)	$M_{BC} = M_{BA}$ (Equation 47)	$M_{CB} = M_{CD}$ (Equation 48)	$M_{DC} = M_{DE}$ (Equation 49)
.1	$-.092\ PL_2$	$-.006\ PL_2$	$-.044\ PL_2$	$-.025\ PL_2$	$+.010\ PL_2$
.2	$-.164$ "	$-.026$ "	$-.081$ "	$-.052$ "	$+.021$ "
.3	$-.203$ "	$-.065$ "	$-.107$ "	$-.081$ "	$+.033$ "
.4	$-.202$ "	$-.122$ "	$-.118$ "	$-.109$ "	$+.044$ "
.5	$-.163$ "	$-.182$ "	$-.112$ "	$-.130$ "	$+.052$ "
.6	$-.106$ "	$-.219$ "	$-.093$ "	$-.135$ "	$+.054$ "
.7	$-.055$ "	$-.214$ "	$-.069$ "	$-.120$ "	$+.048$ "
.8	$-.021$ "	$-.169$ "	$-.044$ "	$-.090$ "	$+.036$ "
.9	$-.004$ "	$-.094$ "	$-.021$ "	$-.048$ "	$+.019$ "

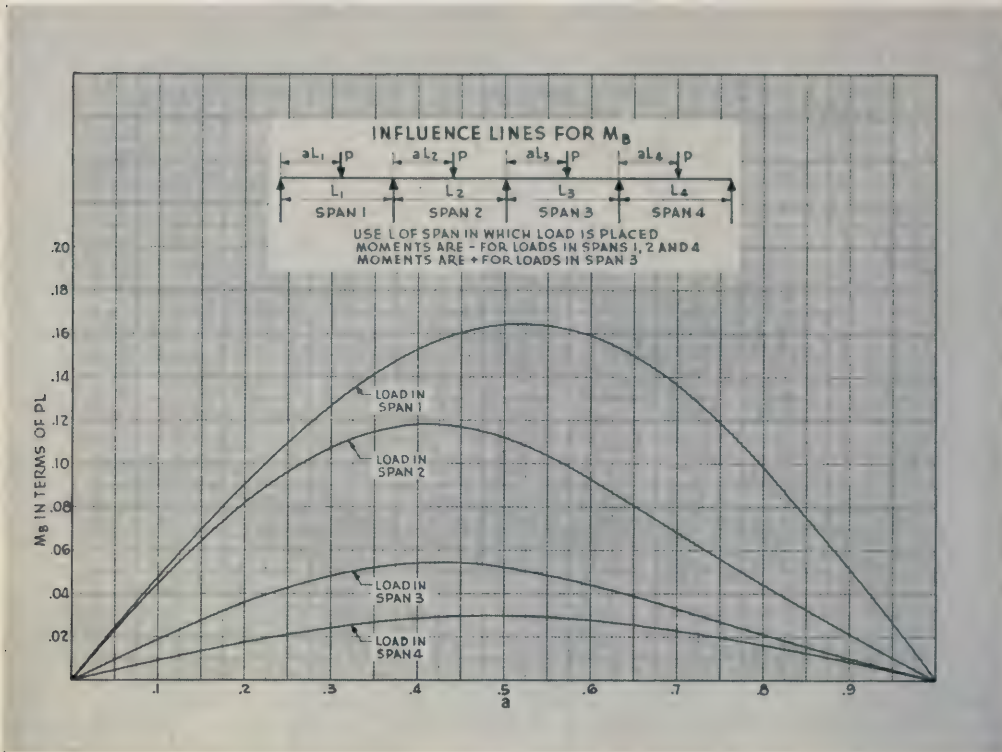

INFLUENCE LINES FOR M_B
aL_1 P
aL_2 P
aL_3 P
aL_4 P
L_1
L_2
L_3
L_4
SPAN 1
SPAN 2
SPAN 3
SPAN 4
USE L OF SPAN IN WHICH LOAD IS PLACED
MOMENTS ARE - FOR LOADS IN SPANS 1, 2 AND 4
MOMENTS ARE + FOR LOADS IN SPAN 3
LOAD IN SPAN 1
LOAD IN SPAN 2
LOAD IN SPAN 3
LOAD IN SPAN 4
M_B IN TERMS OF PL
a

Fig. 49

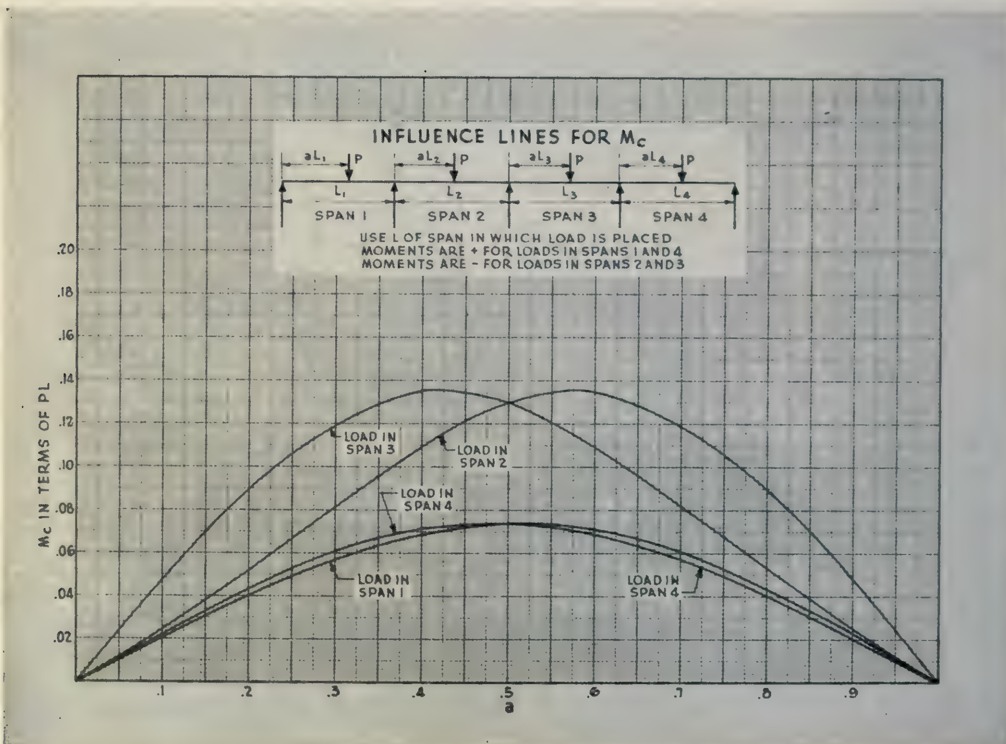

INFLUENCE LINES FOR M_C
aL_1 P
aL_2 P
aL_3 P
aL_4 P
L_1
L_2
L_3
L_4
SPAN 1
SPAN 2
SPAN 3
SPAN 4
USE L OF SPAN IN WHICH LOAD IS PLACED
MOMENTS ARE + FOR LOADS IN SPANS 1 AND 4
MOMENTS ARE - FOR LOADS IN SPANS 2 AND 3
LOAD IN SPAN 3
LOAD IN SPAN 2
LOAD IN SPAN 4
LOAD IN SPAN 1
LOAD IN SPAN 4
M_C IN TERMS OF PL
a

Fig. 50

Influence lines for $M_{BA} = M_{BC}$, and $M_{CB} = M_{CD}$ can now be drawn, as shown in Figs. 49 and 50. Due to symmetry, Spans 1 and 4, and Spans 2 and 3 are the same; therefore, influence lines for moments at the first two supports suffice. Had the bridge been unsymmetrical it would have been necessary to compute and plot influence lines for all three interior supports for loads in all four spans.

Step 3. Maximum Live Load Moments at Critical Sections

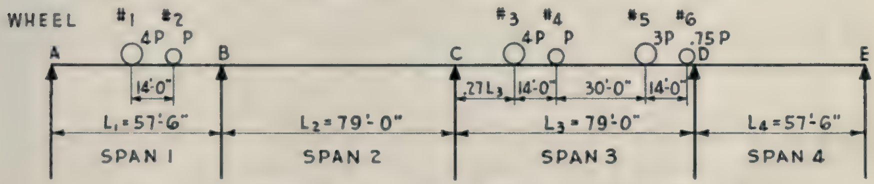


Fig. 51. Loading for maximum positive moment in Span 1.

For maximum positive moment in Span 1, first place loads in Span 3 so as to produce maximum $+M_B$ which is found to be when Wheel 3 is at $0.27L_3$ as shown in Fig. 51. From influence line for moment at B for load in Span 3, Fig. 49:

$$M_{BA} = (0.045 \times 4P + 0.054 \times P + 0.019 \times 3P)^{*}\, 79 = 23.0P$$

For Wheel 1 at $0.33L_1$, and Wheel 2 at $0.57L_1$

$$M_{BA} = -(0.135 \times 4P + 0.162 \times P)\, 57.5 = -40.4P$$

and the positive moment under Wheel 1 is

$$M_{.33L_1} = 3.11P \times 0.33 \times 57.5 - 0.33\,(40.4 - 23.0)P = 53.3P \qquad (50)$$

Similarly, $M_{.35L_1} = 53.9P$ (51)

$$M_{.38L_1} = 54.9P \qquad (52)$$

$$\text{Impact factor} = \frac{50}{125 + 57.5 + 79} = 0.19$$

Then for a 9-ft. girder spacing:

$$P^{**} = 1.19 \times 8{,}000 \times 0.9 = 8{,}575 \text{ lb.}$$

so

$$M_{.33L_1} = 53.3 \times 8{,}575 = 457 \text{ ft.kips} \qquad (53)$$
$$M_{.35L_1} = 53.9 \times 8{,}575 = 462 \text{ ft.kips} \qquad (54)$$
$$M_{.38L_1} = 54.9 \times 8{,}575 = 471 \text{ ft.kips} \qquad (55)$$

Maximum negative moments at supports B and C and maximum positive moment in Span 2 are obtained in a similar manner by placing loads

---

*Wheel 6, in this case, comes directly over support D; hence produces zero moment at B.

**P is the total load on the front axle distributed over a 10-ft. traffic lane and increased by the impact factor, and since in this case the girders are 9 ft. apart, 0.9 of the load on one lane is carried by a single girder.

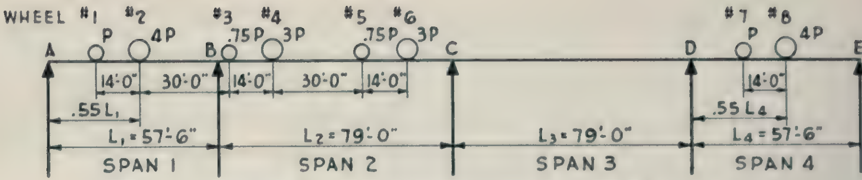


Fig. 52. Loading for maximum negative moment at support *B*.

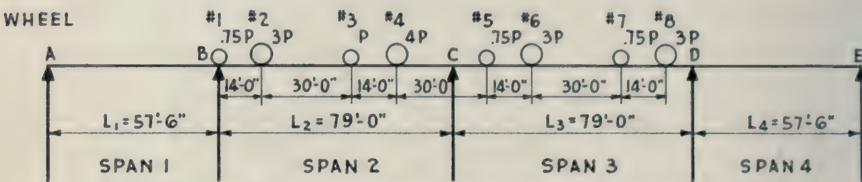


Fig. 53. Loading for maximum negative moment at support *C*.

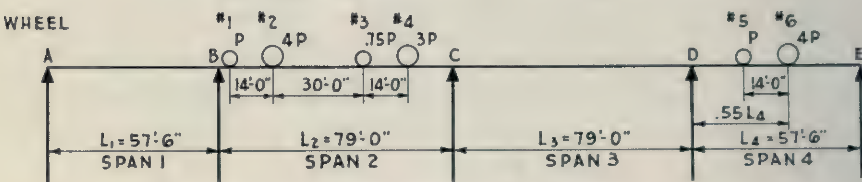


Fig. 54. Loading for maximum positive moment in Span 2.

as shown in Figs. 52, 53 and 54, respectively. The moments at these critical sections will be found to be as follows:

$$M_{BA} = M_{BC} = -776 \text{ ft.kips} \tag{56}$$
$$M_{CB} = M_{CD} = -864 \text{ ft.kips} \tag{57}$$
$$M_{.45L_2} = 440 \text{ ft.kips} \tag{58}$$
$$M_{.48L_2} = 439 \text{ ft.kips} \tag{59}$$
$$M_{.50L_2} = 435 \text{ ft.kips} \tag{60}$$

Step 4. Maximum Dead Load Moments

Substitute the fixed end moments due to the yet undetermined uniform load w and the haunch loads W in the formulas in Table I, considering each span loaded successively.

For uniform load w in Span 1 from Fig. 16:

$$M^F_{AB} = -0.059wL_1^2 \qquad M^F_{BA} = -0.141wL_1^2$$

and by Equation 1, page 17:

$$M_1 = -0.141wL_1^2 - 0.888 \times 0.059wL_1^2 = -0.193wL_1^2 = -639w.$$

The coefficients of M_1 determined in Equations 44 to 46 may now be multiplied by the value of M_1 to obtain the final distributed moments as:

$$M_{BA} = M_{BC} = -0.536 \times 639w = -342w$$
$$M_{CB} = M_{CD} = +0.240 \times 639w = +153w$$
$$M_{DC} = M_{DE} = -0.096 \times 639w = -\ \ 62w$$

Similarly for haunch load at B Span 1 from Fig. 17:

$$M^F_{AB} = -0.001 W_{BA} L_1^2 \qquad M^F_{BA} = -0.018 W_{BA} L_1^2$$

so

$$M_1 = -0.018 W_{BA} L_1^2 - 0.888 \times 0.001 W_{BA} L_1^2 = -62.4 W_{BA}$$

and

$$\begin{aligned} M_{BA} &= M_{BC} = -33.4 W_{BA} \\ M_{CB} &= M_{CD} = +15.0 W_{BA} \\ M_{DC} &= M_{DE} = -\ \ 6.0 W_{BA} \end{aligned}$$

For loads in Span 2:

$$\begin{aligned} M_{BA} &= M_{BC} = -434w - 52.2 W_{BC} - 28.3 W_{CB} \\ M_{CB} &= M_{CD} = -497w - 33.9 W_{BC} - 57.2 W_{CB} \\ M_{DC} &= M_{DE} = +200w + 13.6 W_{BC} + 22.9 W_{CB} \end{aligned}$$

For loads in Span 3—by symmetry:

$$\begin{aligned} M_{BA} &= M_{BC} = +200w + 13.6 W_{BC} + 22.9 W_{CB} \\ M_{CB} &= M_{CD} = -497w - 33.9 W_{BC} - 57.2 W_{CB} \\ M_{DC} &= M_{DE} = -434w - 52.2 W_{BC} - 28.3 W_{CB} \end{aligned}$$

For loads in Span 4—by symmetry:

$$\begin{aligned} M_{BA} &= M_{BC} = -\ \ 62w -\ \ 6.0 W_{BA} \\ M_{CB} &= M_{CD} = +153w + 15.0 W_{BA} \\ M_{DC} &= M_{DE} = -342w - 33.4 W_{BA} \end{aligned}$$

Noting that $W_{BA} = W_{BC}$, the total dead load moments at supports will be obtained by adding the moments due to loads in each span:

$$M_{BA} = M_{BC} = -638w - 77.9 W_{BC} - \ \ 5.4 W_{CB} \qquad (61)$$

$$M_{CB} = M_{CD} = -688w - 38.0 W_{BC} - 114.4 W_{CB} \qquad (62)$$

$$M_{DC} = M_{DE} = -638w - 77.9 W_{BC} - \ \ 5.4 W_{CB} \qquad (63)$$

Step 5. Select Girder Spacing, Determine Girder Width and Design Slab

In this problem a 9-ft. spacing was selected arbitrarily in Step 3 to complete the determination of live load moments.

From the formula in Step 5, "Design Procedure for T-girders", $b' = 0.0025\ \sqrt{108} \times 57.5 \times 12 = 17.9$ in.—use 16.5 or 20.25 in., which provides for 4 and 5 bars respectively in one layer in the stem for positive moment. Since the same thickness slab is required in either case, the 16½-in. width will be used.

Live load plus impact moment, from Fig. 45 $= 0.31 \times 16{,}000 = 4{,}960$ ft.lb.

Dead load moment (7-in. slab assumed) $= \dfrac{ws^2}{10} = 0.1 \times 87.5 \times 6.62^2 = \ \ 380$ ft.lb.

5,340 ft.lb.

$$d, \text{ required} = 2 + \sqrt{\frac{5{,}340}{208}} = 7.1 \text{ in., say 7 in.}$$

No wearing surface will be required.

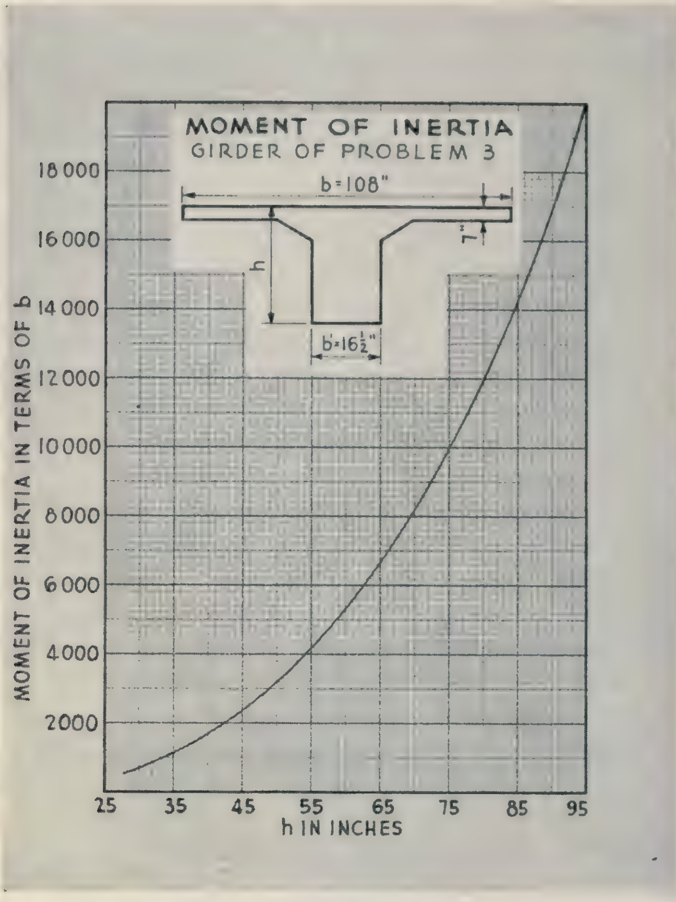


Fig. 55

Step 6. Assume Trial Depth at Supports

Fig. 55 is the moment of inertia curve for the gross section of the deck taken from center to center between girders and for an over-all depth of girder ranging from approximately $0.04L_1$ to $0.12L_1$. Values required for plotting the curve were obtained by entering Fig. 46 with $\frac{b'}{b} = \frac{16.5}{108} = 0.153$ and various ratios of $\frac{t}{h}$.

From Fig. 47 a trial depth at support B is obtained as 71 in. With this value enter the moment of inertia curve just plotted, Fig. 55, and obtain $I_s = 8{,}600b$. The moment of inertia at the center of Span 1 is therefore:

$$I_c = \frac{I_s}{(1 + r_B)^3} = \frac{8.600b}{(1 + 1.3)^3} = 707b$$

and by reentering Fig. 55 the corresponding depth of girder at the center

is obtained as:

$$h_c = 30.0 \text{ in.}$$

at support C

$$I_s = I_c\,(1 + r_C)^3 = 707b\,(1 + 1.5)^3 = 11{,}040b$$

for which at support C

$$h_s = 77.5 \text{ in.}$$

Step 7. Check Assumed Sections

From the trial dimensions, w = 1,240 lb.; W_{BA} = W_{BC} = 705 lb.; and W_{CB} = 816 lb. With these values the total dead load moments are readily found from Equations 61 to 63:

For $M_{BA} = M_{BC}$

Dead load moment:

$$-638 \times 1{,}240 - 77.9 \times 705 - 5.4 \times 816 = -850 \text{ ft.kips}$$

Live load moment (Equation 56): $= -776$ ft.kips

$$\text{Total } M_{BA} = M_{BC} = -1{,}626 \text{ ft.kips}$$

Stress analysis shows a depth h_B = 72 in. will be required at support B instead of the assumed depth of 71 in. which will increase the weight of the various sections to be used in subsequent computations to w = 1,248 lb.; W_{BA} = W_{BC} = 714 lb.; W_{CB} = 825 lb.*

The positive moment in Span 1 at $0.33L_1$ will be:

Dead load moment: $\dfrac{1{,}248}{2} \times 0.33 \times 0.67 \times 57.5^2 \quad = \quad 455$ ft.kips

$$\frac{714}{3} \times \frac{57.5}{2} \times \frac{1}{8} \times 0.33 \times 57.5 = 16 \text{ ft.kips}$$

(*See Equation 61*) $\quad 0.33\,(-638 \times 1{,}248 - 77.9 \times 714 - 5.4 \times 825) = -283$ ft.kips

188 ft.kips

Live load moment (Equation 53): $= 457$ ft.kips

$$\text{Total } + M_{.33L_1} = 645 \text{ ft.kips}$$

Similarly

$M_{.35L_1}$ = 647 ft.kips (maximum moment in Span 1)

$M_{.38L_1}$ = 647 ft.kips

At support C, $M_{CB} = M_{CD} = -1{,}846$ ft.kips and the positive moment in Span 2 will be:

$M_{.45L_2}$ = 605 ft.kips (maximum moment in Span 2)

$M_{.48L_2}$ = 602 ft.kips

$M_{.50L_2}$ = 588 ft.kips

A check of the stresses should now be made at the critical sections. At the 0 $35L_1$ point, with an effective depth of about 22.5 in. it will be found that f_c = 990 p.s.i. and f_s = 17,800 p.s.i., with fourteen 1¼-in. square bars. The low stress in the concrete shows that a larger r value could have been used but the number of bars required necessitates placing them in four layers so a shallower depth would be impracticable.

*The difference between the required and assumed depth in this example is so small that no appreciable error would result if the weights were not corrected, but the adjusted weights have been used here as an illustration of procedure.

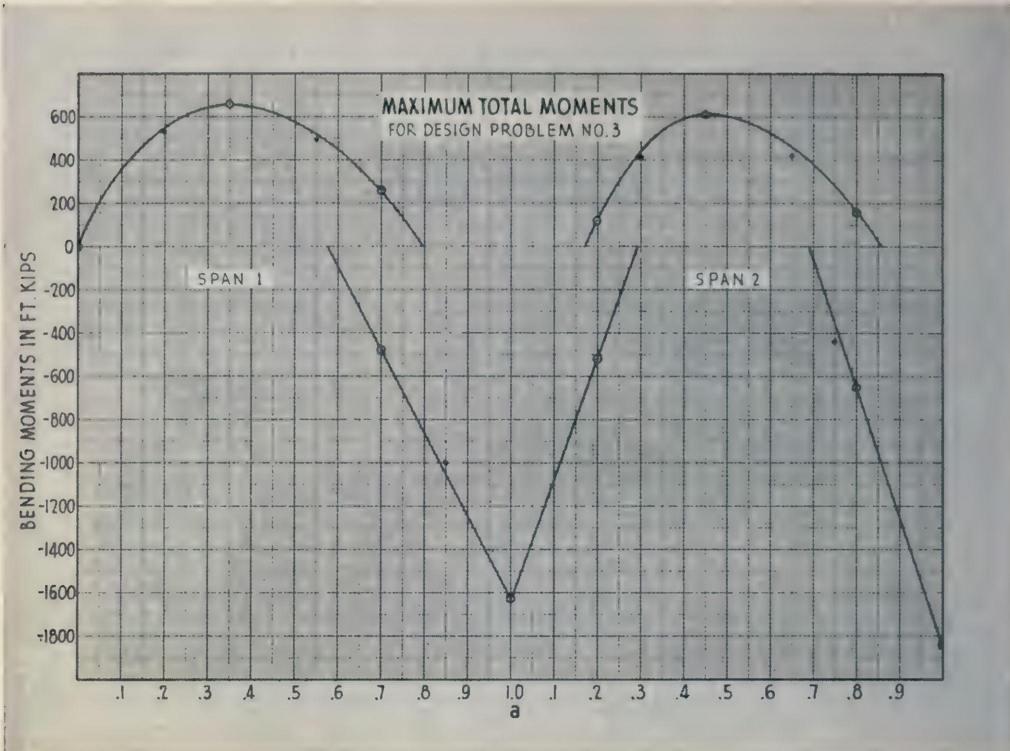

MAXIMUM TOTAL MOMENTS
FOR DESIGN PROBLEM NO. 3
BENDING MOMENTS IN FT. KIPS
SPAN 1
SPAN 2
a

Fig. 56

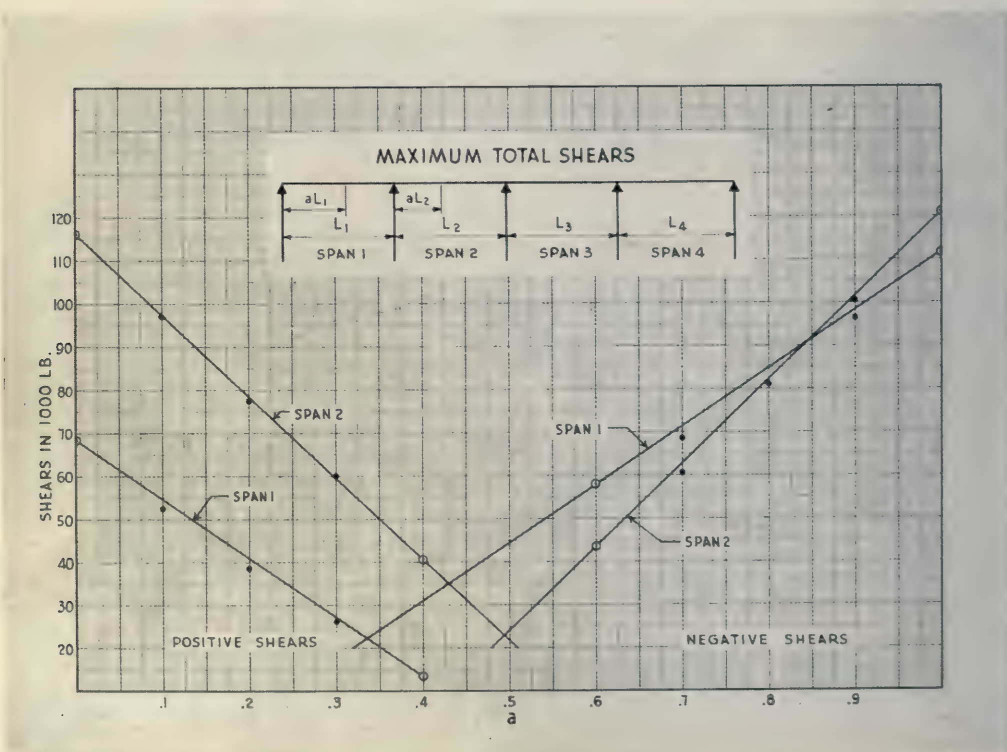

MAXIMUM TOTAL SHEARS
SPAN 1
SPAN 2
SPAN 3
SPAN 4
SHEARS IN 1000 LB.
POSITIVE SHEARS
NEGATIVE SHEARS
a

Fig. 57

Step 8. Curve of Maximum Moments

Having determined satisfactory values of h_c and h_s, the total live and dead load moments may be computed at a sufficient number of sections so the moment curves can be drawn. The procedure for determining the moments is the same as that already demonstrated and the curves are drawn as explained under Problem No. 1 so will not be repeated. It will be convenient to use Fig. 17a for determination of dead load moments at intermediate sections between supports due to parabolic loads. Fig. 56 shows the completed curves.

Step 9. Maximum Total Shears

As a final operation the shears at the supports and at intermediate points must be determined and curves of maximum shear drawn as explained on page 64. Maximum live and dead load shears are combined in Table XIV and the curves are shown in Fig. 57. Note that supplementary computed values indicated thus, •, fall sufficiently close for all practical purposes to the curves drawn as straight lines through two points near their extremities.

TABLE XIV—Maximum Shears at Supports and Intermediate Sections Required for Drawing Shear Curves

Span	Section	Dead load shear kips	Live load and impact shear kips	Total shear kips
1	Support *A*	+21.9	+46.5	+ 68.4
	$0.4L_1$	− 6.8	+20.5	+ 13.7
	$0.6L_1$	−21.3	−36.5	− 57.8
	Support *B*	−56.8	−54.7	−111.5
2	Support *B*	+57.3	+58.9	+116.2
	$0.4L_2$	+ 8.6	+31.9	+ 40.5
	$0.6L_2$	−11.3	−32.2	− 43.5
	Support *C*	−61.5	−59.5	−121.0

Step 10. Selection and Arrangement of Reinforcement

Fig. 58a shows an arrangement of reinforcement which satisfies the requirements of maximum moment and shear indicated by the curves, Figs. 56 and 57.

Fig. 58b shows the main longitudinal bars distributed in the slab in proportion to the tensile stresses to be resisted by the slab; this arrangement of girder reinforcement is discussed in detail on page 106. When the tensile steel over supports is so distributed, the spacing of the bars will usually be such that W-stirrups may be used without having to weave bars into place.

In the curved soffit portions of the girders, the stirrups will be of variable heights. The single unit W-stirrup should be more economical than a stirrup made of two or more pieces and can be more easily placed. In girders 16 in.

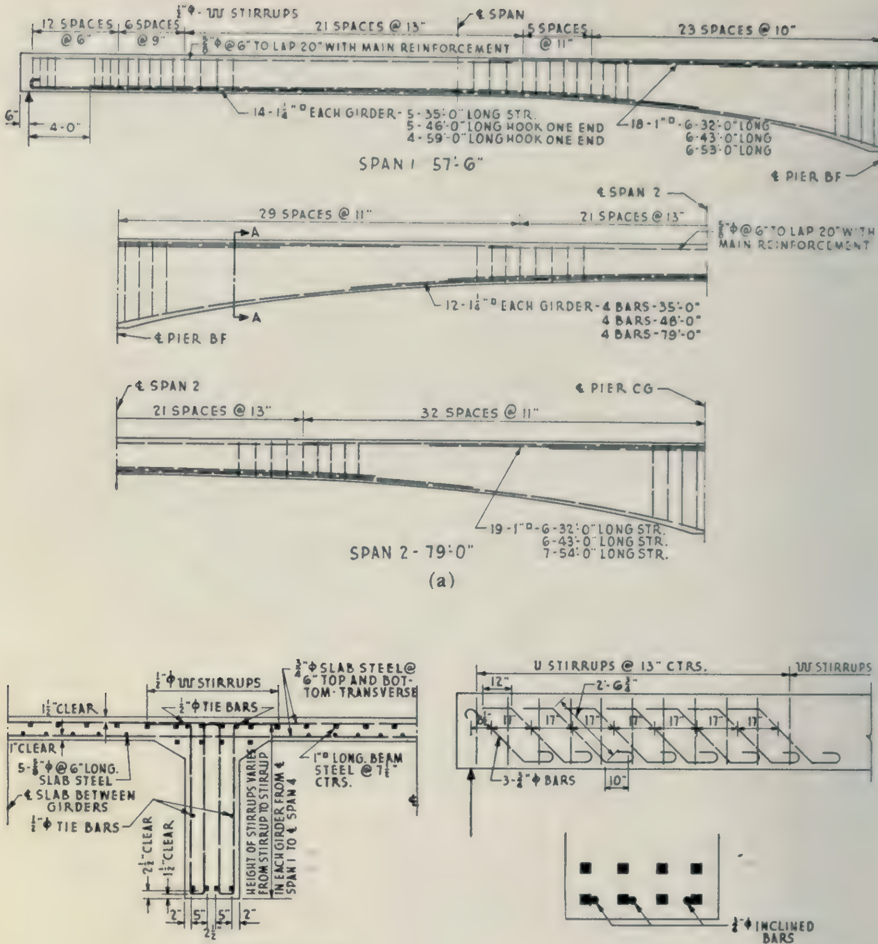


Fig. 58. Arrangement of reinforcement in deck of bridge of Design Problem No. 3.

or more in width, a W-stirrup gives a much better stress-resisting arrangement of metal than does the U-stirrup.

In lieu of the close spacing of vertical stirrups at free ends of girders of Spans 1 and 4, the arrangement of separate inclined bars shown in the end detail drawing Fig. 58c may be preferable and would be more effective. This scheme of bent-up bars is more economical and practical than bending up the 1¼-in. girder bars.

The ⅝-in. round bars shown in the bottom of the slab are required to resist longitudinal stresses set up by concentrated loads; 75 per cent of the transverse steel is usually provided in the longitudinal direction in the positive moment area*.

*See bulletin, *Distribution of Wheel Loads and Design of Reinforced Concrete Bridge Floor Slabs*, published July, 1934 by Bureau of Public Roads.

Section VII—Deflections

Deflection of concrete bridges due to their own weight is small, and for ordinary span bridges is neglected. The demand for smooth-riding roadway surfaces makes it necessary, however, to construct longer span bridges so that the deck will have the desired final grade after deflection has taken place. Deviations from desired final grade after removal of falsework result from settlement of supports and deflection of the deck between supports. The settlement of supports is usually very small, and if the supports are properly designed the settlement will be practically the same for all and therefore will not affect the riding quality of the bridge roadway. Compensation should be made for deflection of the deck by adding the computed anticipated deflections to the desired final grade line and so constructing the falsework that the roadway surface will be finished to these new elevations. In general, the grade line before falsework is struck will be above the final grade line except close to supports in short spans where the effect of a long adjoining span may cause an uplift when forms are removed. For smooth riding, deflections should be computed to the nearest hundredth of an inch at every tenth point of each span.

It is impractical to attempt to compensate for deflections due to live load because of the changing position and the relatively small magnitude of the load. On the other hand, the deflection due to dead load is definite and always present. Dead load deflection may not be objectionable for short spans and may be neglected for spans up to about 40 ft., but for longer spans it should be taken into account. A method is presented here by which dead load deflections can be obtained.

Computation of the deflection of members having a variable moment of inertia becomes a laborious task if it must be done by solution of fundamental deflection formulas. In order to facilitate the determination of the deflections of bridge decks whose moments of inertia vary as shown in Fig. 4, page 13, curves are presented here (*see Figs. 59 to 63*) giving deflection coefficients for uniform load, parabolically increasing load, and moment at end of span*. Although the curves were prepared primarily for determination of deflection in bridge decks, they can be used for any beam, girder or slab whose moment of inertia varies approximately as that shown in Fig. 4 mentioned above.

The procedure for obtaining the deflection at any point in a span consists simply of entering the curves with the proper values of r_A, r_B and a and reading the deflection coefficient which must be multiplied by the factor indicated in the respective curve. This factor depends upon whether effect of uniform load, parabolically increasing load, or end moment is being determined.

It will be noted that the modulus of elasticity of the concrete is involved in each expression for deflection. The ultimate accuracy of computed deflections depends upon selecting the proper value of E_c. It is well known that

(*continued on page 89*)

*The derivation of formulas on which these curves are based will be furnished upon request to the Portland Cement Association. See also *Influence Lines Drawn as Deflection Curves*, published by Portland Cement Association.

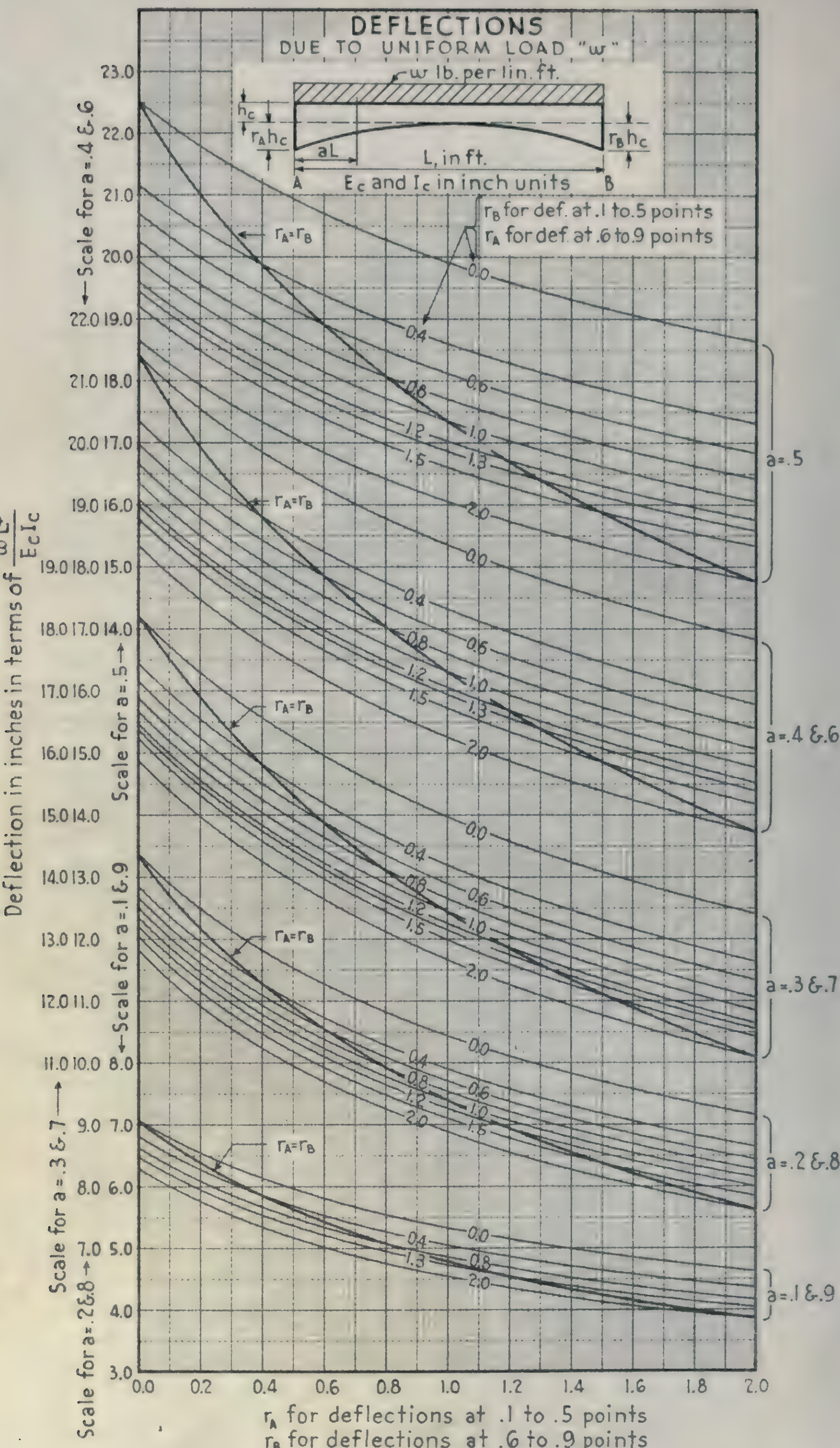

DEFLECTIONS
DUE TO UNIFORM LOAD "w"
w lb. per lin. ft.
h_c
$r_A h_c$
$r_B h_c$
aL
L, in ft.
A
B
E_c and I_c in inch units
r_B for def. at .1 to .5 points
r_A for def. at .6 to .9 points
$r_A = r_B$
a = .5
a = .4 & .6
a = .3 & .7
a = .2 & .8
a = .1 & .9
Deflection in inches in terms of $\frac{wL^4}{E_c I_c}$
Scale for a = .4 & .6
Scale for a = .5
Scale for a = .1 & .9
Scale for a = .3 & .7
Scale for a = .2 & .8
r_A for deflections at .1 to .5 points
r_B for deflections at .6 to .9 points

Fig. 59

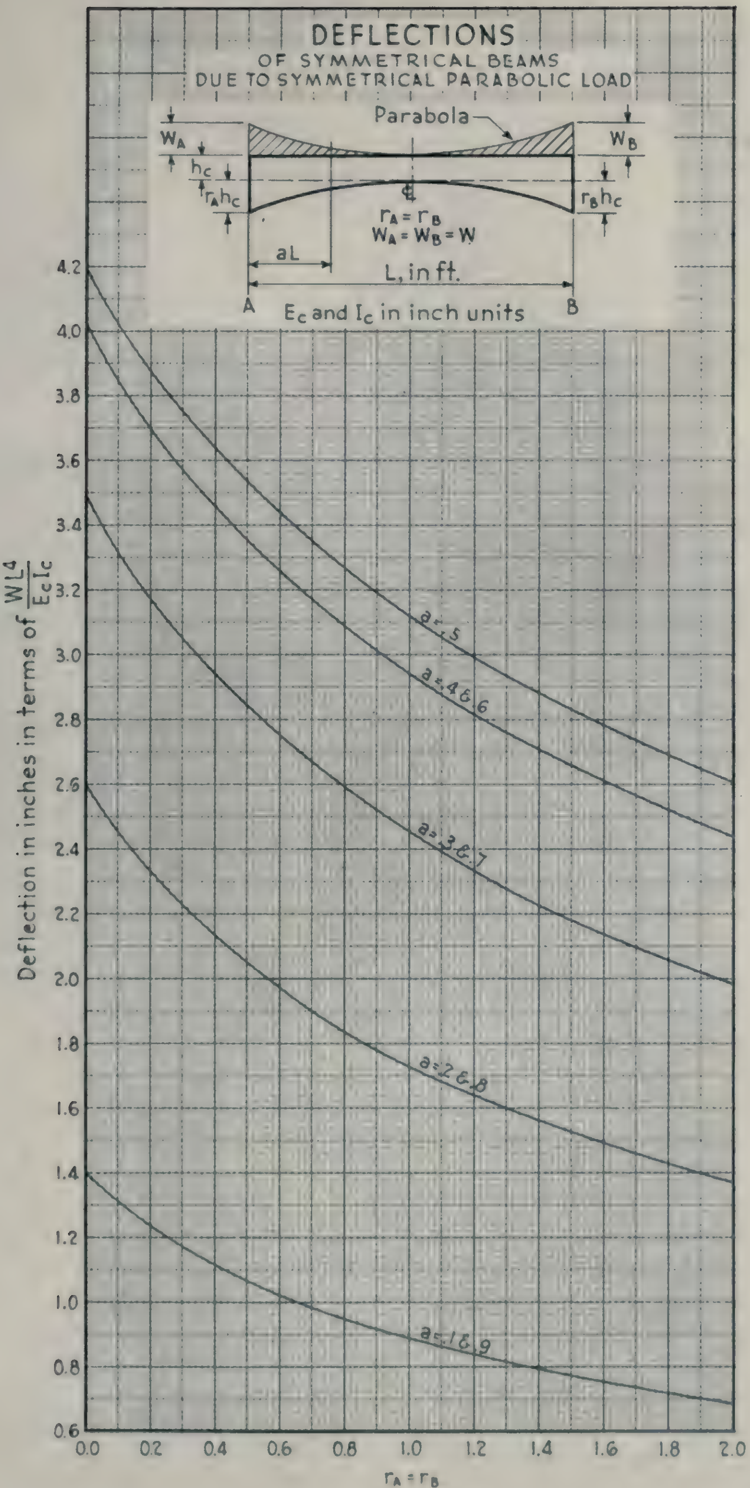

DEFLECTIONS
OF SYMMETRICAL BEAMS
DUE TO SYMMETRICAL PARABOLIC LOAD
Parabola
W_A
W_B
h_c
$r_A h_c$
$r_B h_c$
$r_A = r_B$
$W_A = W_B = W$
aL
L, in ft.
A
B
E_c and I_c in inch units
Deflection in inches in terms of $\frac{WL^4}{E_c I_c}$
a=.5
a=.4&.6
a=.3&.7
a=.2&.8
a=.1&.9
$r_A = r_B$

Fig. 60

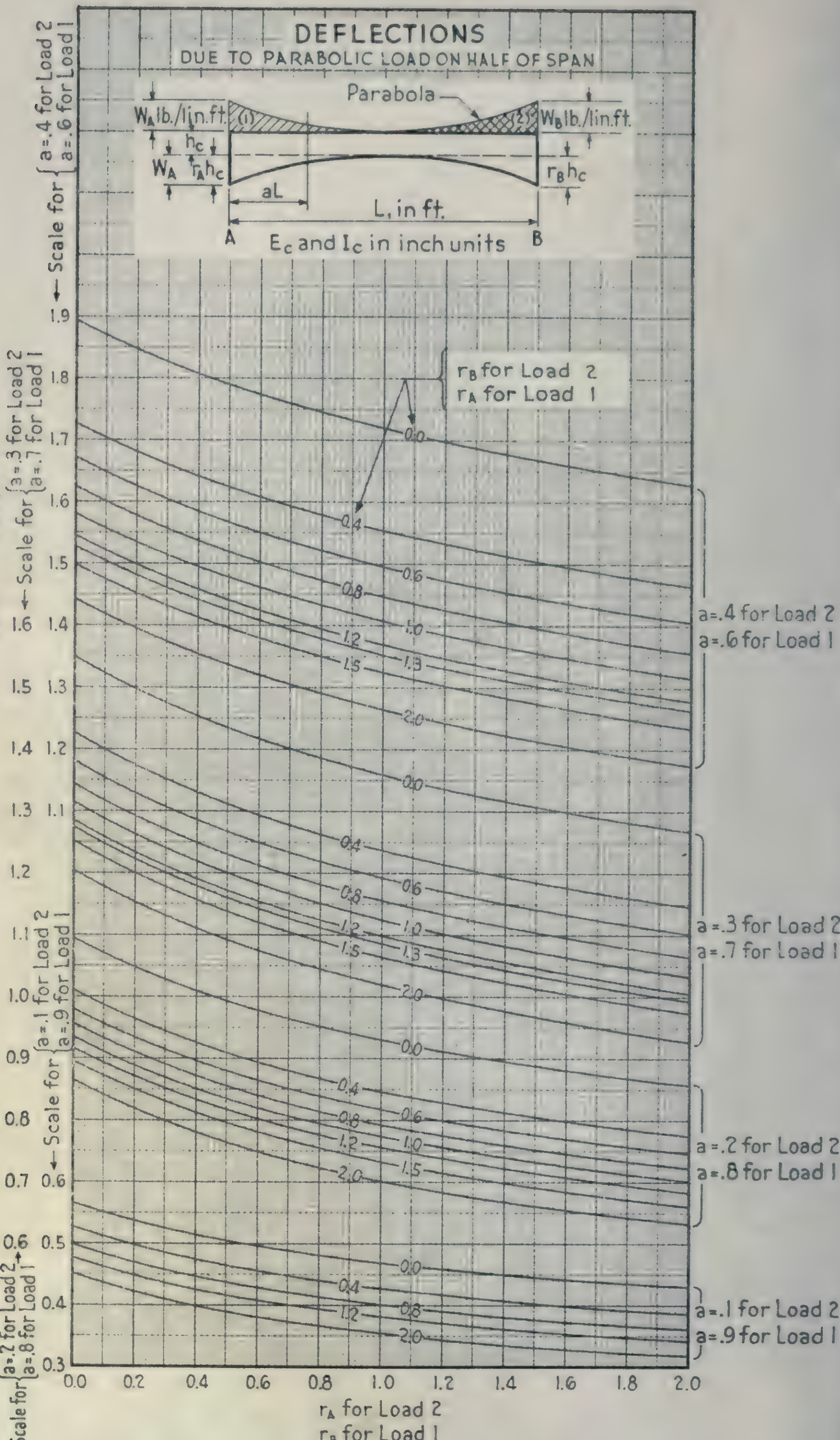

DEFLECTIONS
DUE TO PARABOLIC LOAD ON HALF OF SPAN
Parabola
WA lb./lin.ft.
WB lb./lin.ft.
hc
rA hc
rB hc
aL
L, in ft.
A
B
Ec and Ic in inch units
rB for Load 2
rA for Load 1
a=.4 for Load 2
a=.6 for Load 1
a=.3 for Load 2
a=.7 for Load 1
a=.2 for Load 2
a=.8 for Load 1
a=.1 for Load 2
a=.9 for Load 1
Deflection in inches in terms of WA L4 / Ec Ic or WB L4 / Ec Ic
Scale for a=.4 for Load 2, a=.6 for Load 1
Scale for a=.3 for Load 2, a=.7 for Load 1
Scale for a=.1 for Load 2, a=.9 for Load 1
Scale for a=.2 for Load 2, a=.8 for Load 1
rA for Load 2
rB for Load 1

Fig. 61a

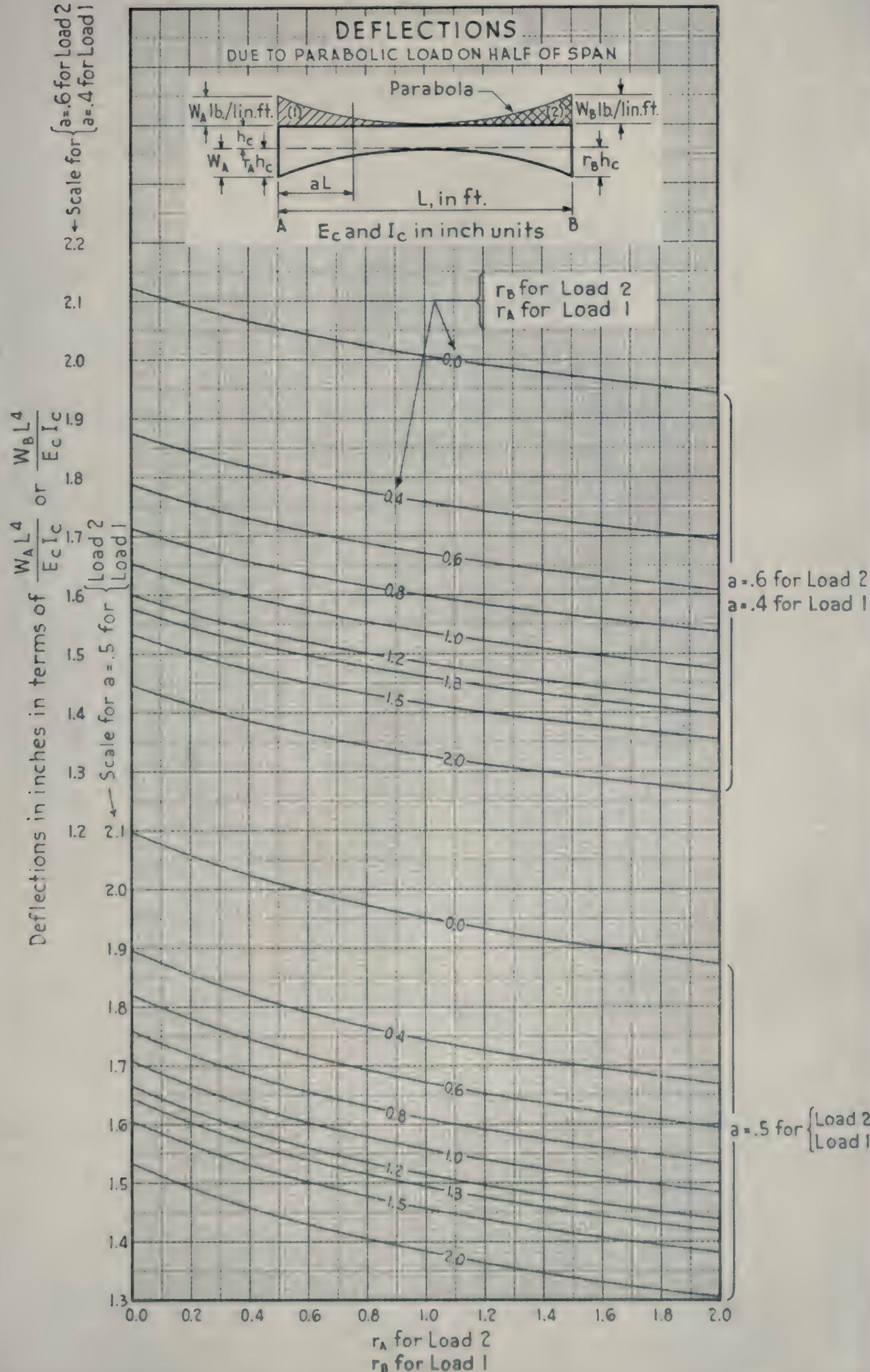
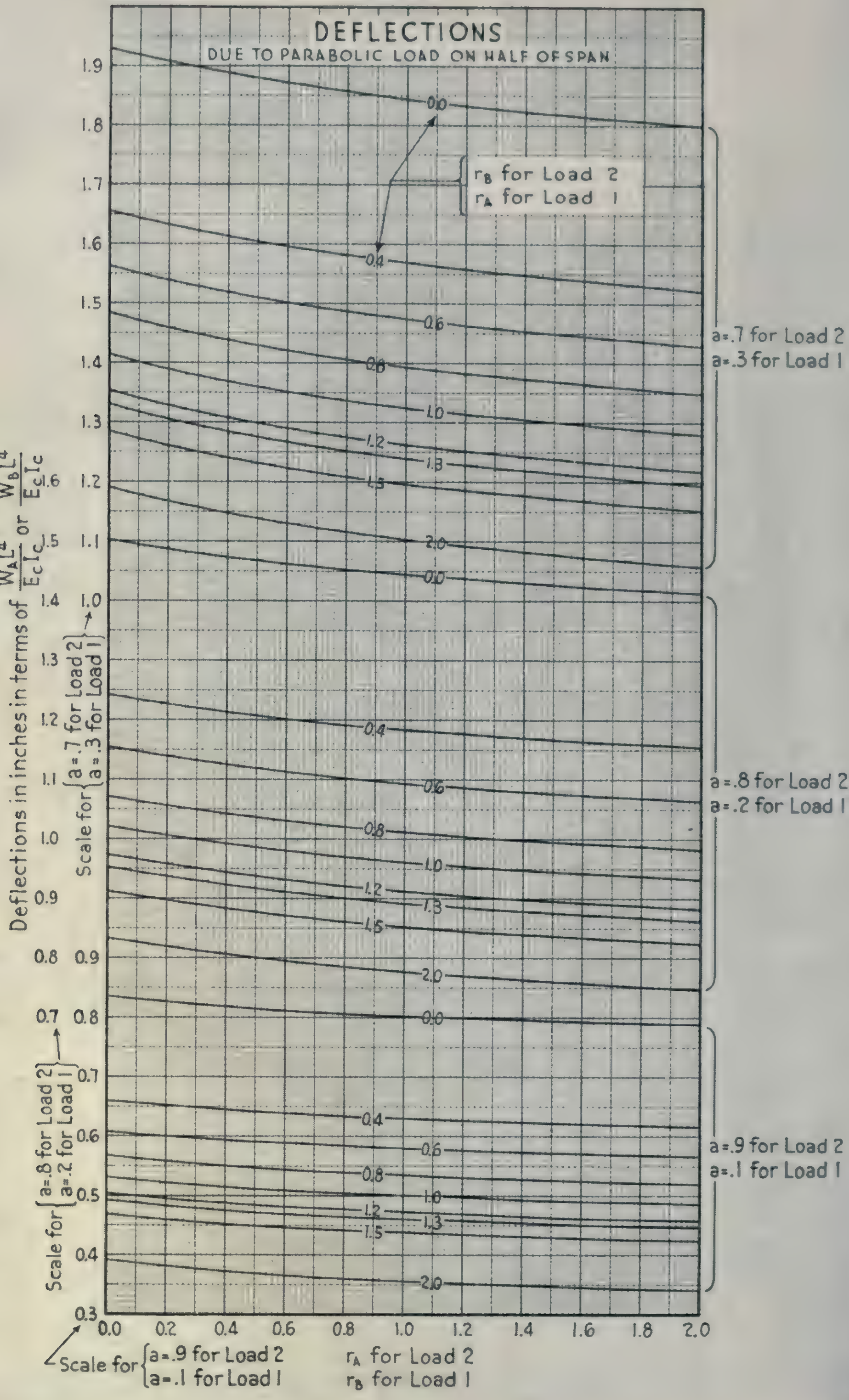

DEFLECTIONS
DUE TO PARABOLIC LOAD ON HALF OF SPAN
Parabola
W_A lb./lin.ft.
W_B lb./lin.ft.
h_c
$r_A h_c$
$r_B h_c$
aL
L, in ft.
A
B
E_c and I_c in inch units
r_B for Load 2
r_A for Load 1
a = .6 for Load 2
a = .4 for Load 1
a = .5 for Load 2, Load 1
r_A for Load 2
r_B for Load 1
Deflections in inches in terms of $\frac{W_A L^4}{E_c I_c}$ or $\frac{W_B L^4}{E_c I_c}$
Scale for a = .6 for Load 2, a = .4 for Load 1
Scale for a = .5 for Load 2, Load 1

Fig. 61b



DEFLECTIONS
DUE TO PARABOLIC LOAD ON HALF OF SPAN
r_B for Load 2
r_A for Load 1
Deflections in inches in terms of $\frac{W_A L^4}{E_c I_c}$ or $\frac{W_B L^4}{E_c I_c}$
Scale for {a=.7 for Load 2, a=.3 for Load 1}
Scale for {a=.8 for Load 2, a=.2 for Load 1}
Scale for {a=.9 for Load 2, a=.1 for Load 1}
a=.7 for Load 2
a=.3 for Load 1
a=.8 for Load 2
a=.2 for Load 1
a=.9 for Load 2
a=.1 for Load 1
r_A for Load 2
r_B for Load 1
0.0 0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6 1.8 2.0
0.0 0.4 0.6 0.8 1.0 1.2 1.3 1.5 2.0

Fig. 61c

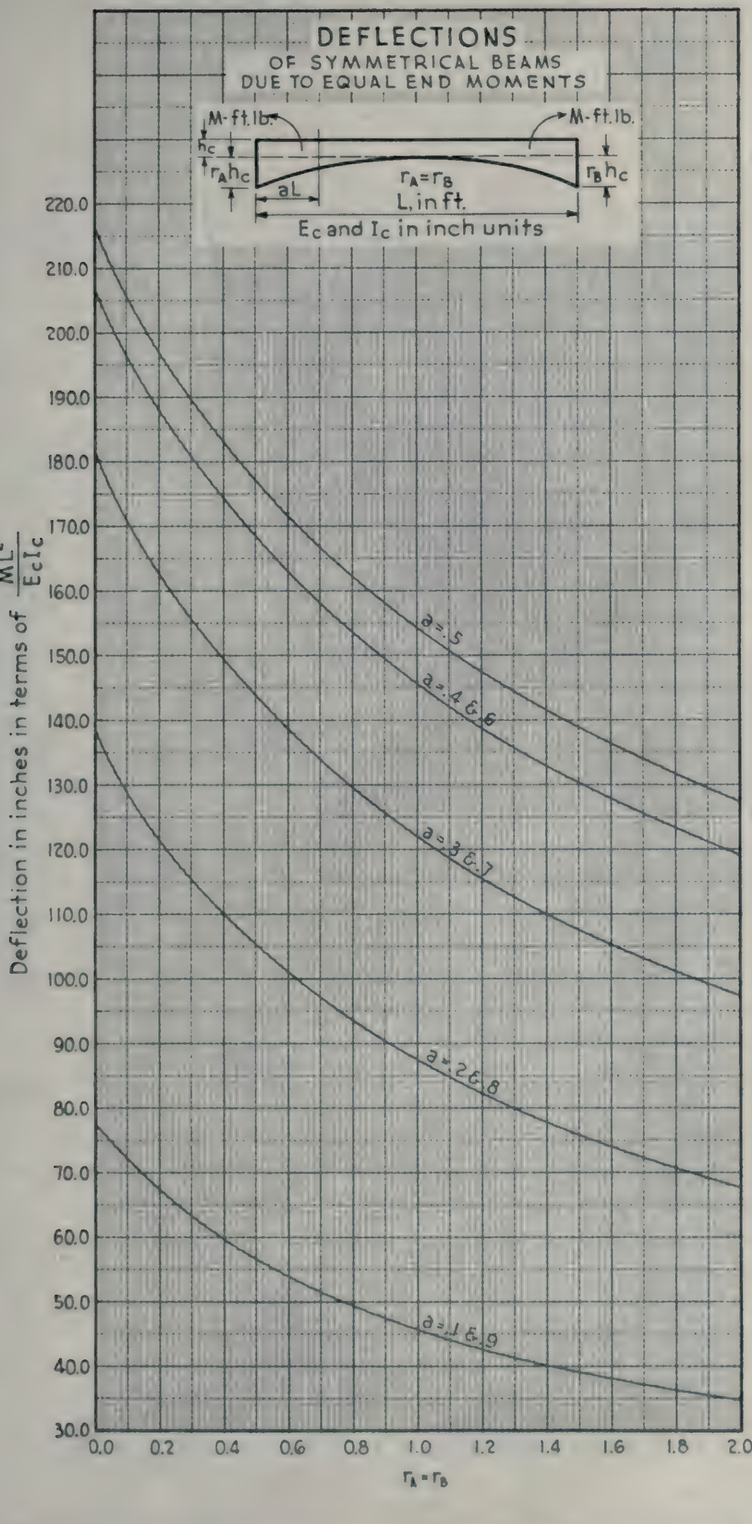

DEFLECTIONS
OF SYMMETRICAL BEAMS
DUE TO EQUAL END MOMENTS
M-ft.lb.
M-ft.lb.
h_c
$r_A h_c$
$r_A = r_B$
$r_B h_c$
aL
L, in ft.
E_c and I_c in inch units
Deflection in inches in terms of $\frac{ML^2}{E_c I_c}$
a=.5
a=.4 & .6
a=.3 & .7
a=.2 & .8
a=.1 & .9
$r_A = r_B$

Fig. 62

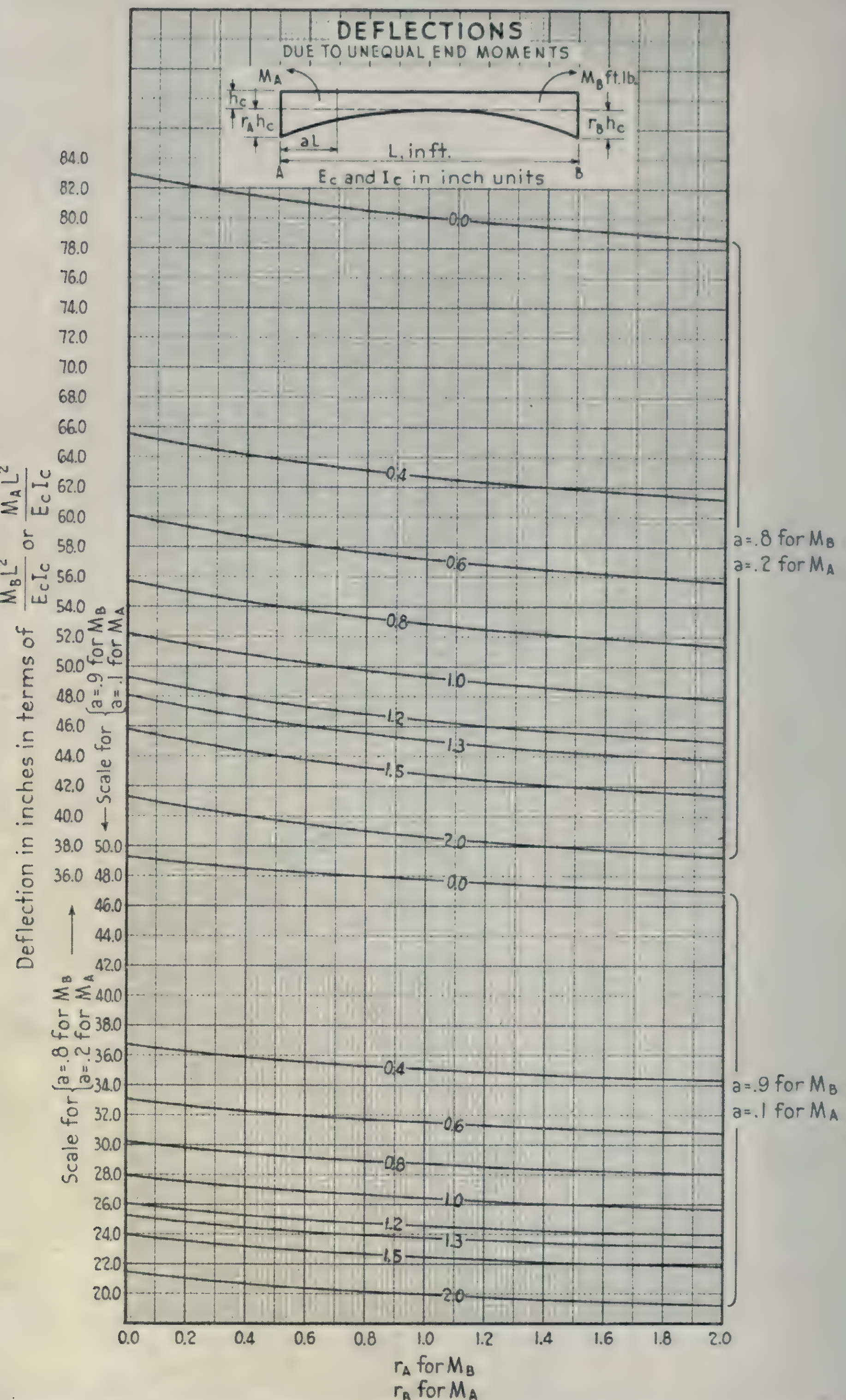

DEFLECTIONS
DUE TO UNEQUAL END MOMENTS
M_A
M_B ft. lb.
h_c
$r_A h_c$
$r_B h_c$
aL
L, in ft.
A
B
E_c and I_c in inch units
Deflection in inches in terms of $\frac{M_B L^2}{E_c I_c}$ or $\frac{M_A L^2}{E_c I_c}$
Scale for a=.9 for M_B, a=.1 for M_A
Scale for a=.8 for M_B, a=.2 for M_A
a=.8 for M_B
a=.2 for M_A
a=.9 for M_B
a=.1 for M_A
r_A for M_B
r_B for M_A

Fig. 63a

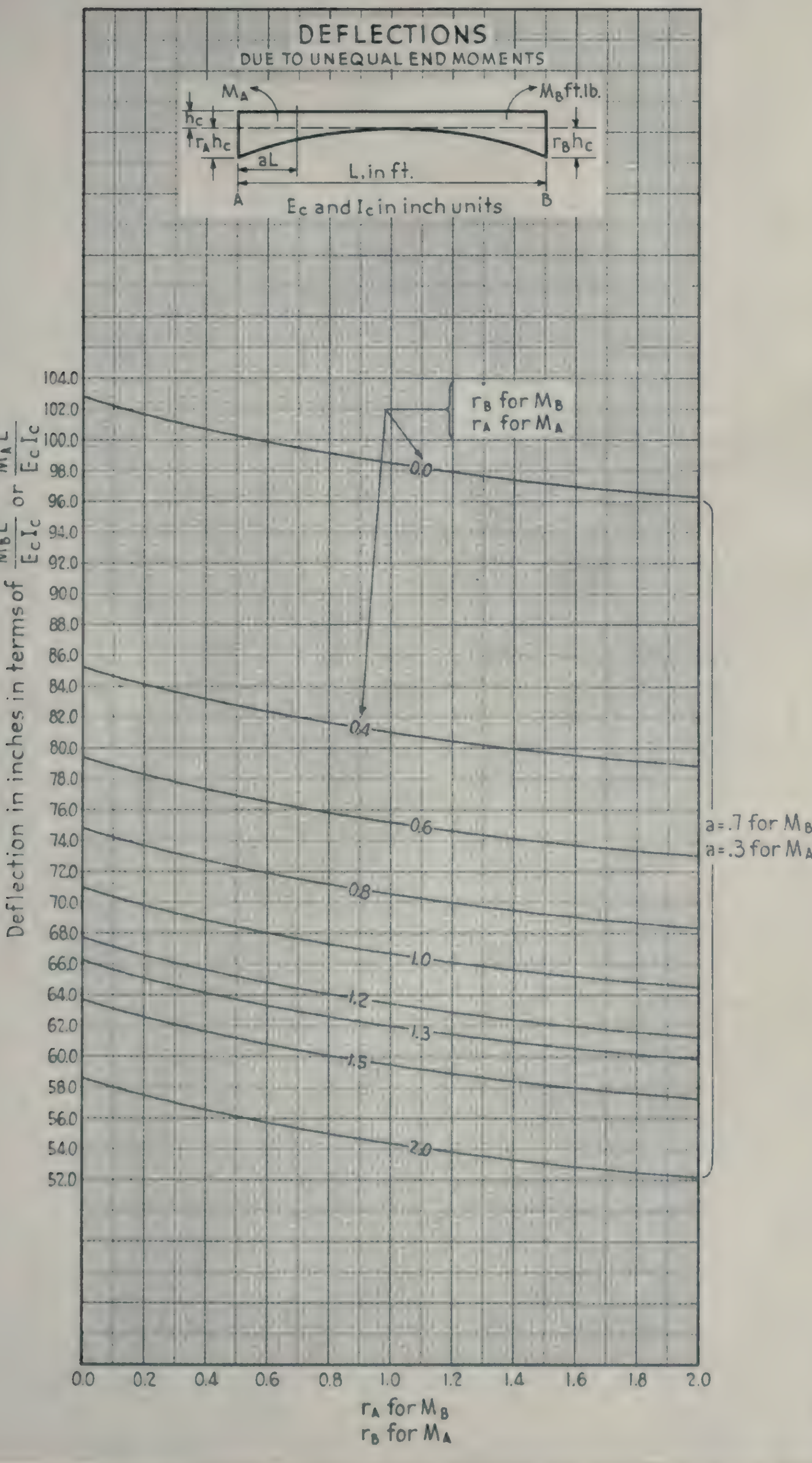

DEFLECTIONS
DUE TO UNEQUAL END MOMENTS
M_A
M_B ft.lb.
h_c
$r_A h_c$
$r_B h_c$
aL
L, in ft.
A
B
E_c and I_c in inch units
r_B for M_B
r_A for M_A
Deflection in inches in terms of $\frac{M_B L^2}{E_c I_c}$ or $\frac{M_A L^2}{E_c I_c}$
a=.7 for M_B
a=.3 for M_A
r_A for M_B
r_B for M_A
0.0
0.4
0.6
0.8
1.0
1.2
1.3
1.5
2.0

Fig. 63b

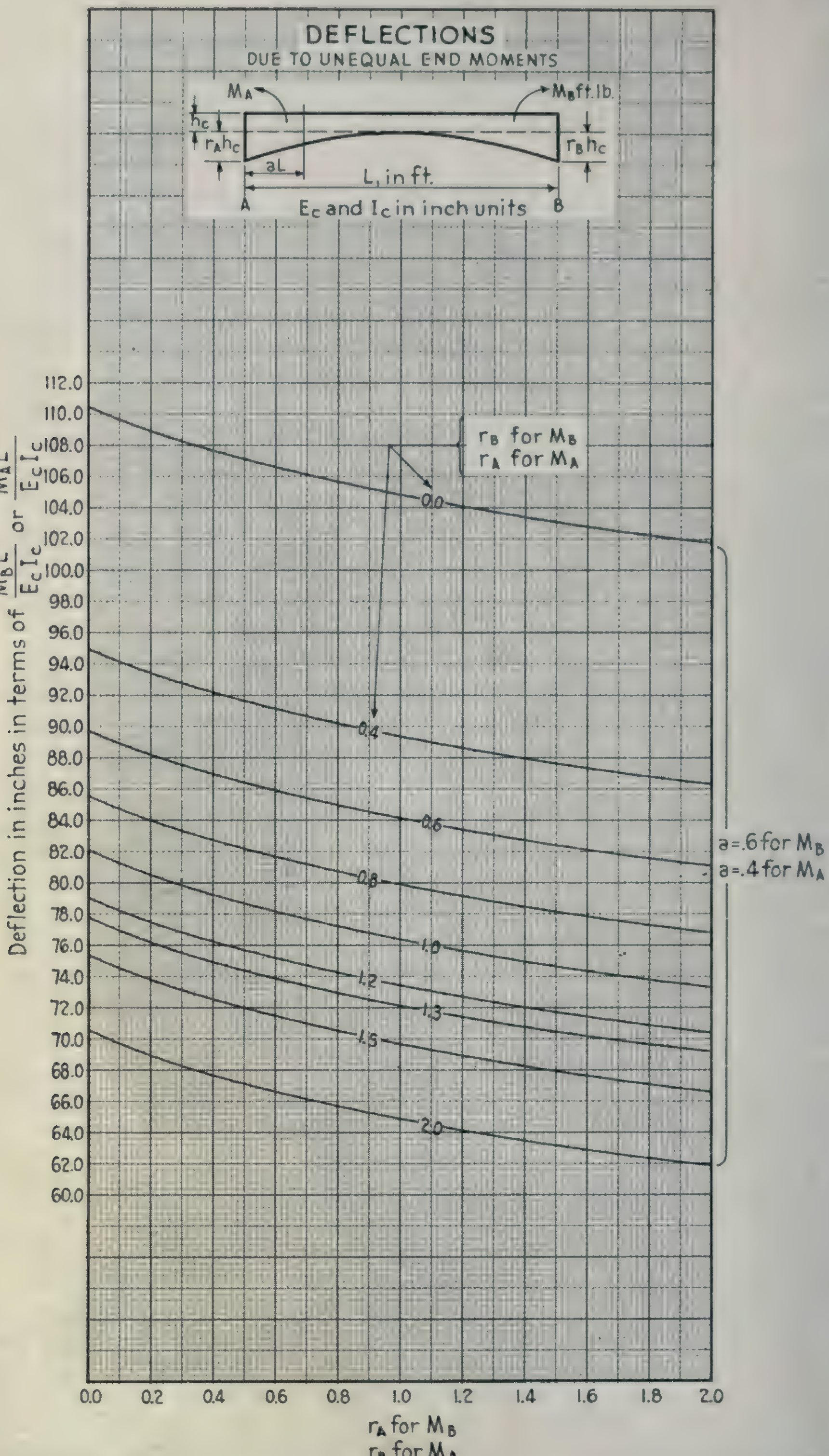

DEFLECTIONS
DUE TO UNEQUAL END MOMENTS
M_A
M_B ft.lb.
h_c
$r_A h_c$
$r_B h_c$
aL
L, in ft.
A
B
E_c and I_c in inch units
r_B for M_B
r_A for M_A
a=.6 for M_B
a=.4 for M_A
Deflection in inches in terms of $\frac{M_B L^2}{E_c I_c}$ or $\frac{M_A L^2}{E_c I_c}$
r_A for M_B
r_B for M_A

Fig. 63c

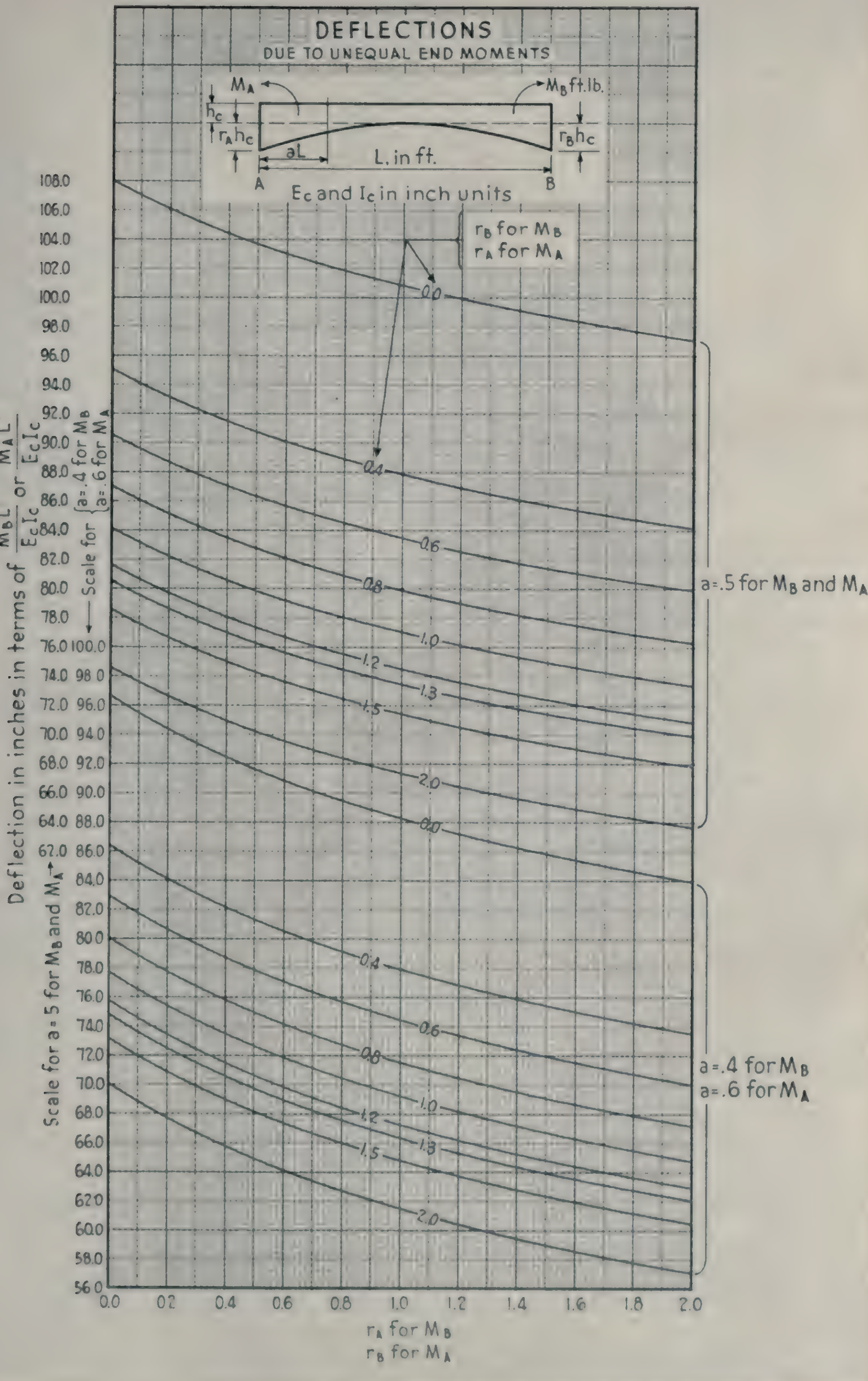

DEFLECTIONS
DUE TO UNEQUAL END MOMENTS
M_A
M_B ft.lb.
h_c
$r_A h_c$
$r_B h_c$
aL
L, in ft.
A
B
E_c and I_c in inch units
r_B for M_B
r_A for M_A
Deflection in inches in terms of $\frac{M_B L^2}{E_c I_c}$ or $\frac{M_A L^2}{E_c I_c}$
Scale for a = .5 for M_B and M_A
Scale for a = .4 for M_B, a = .6 for M_A
a = .5 for M_B and M_A
a = .4 for M_B
a = .6 for M_A
r_A for M_B
r_B for M_A

Fig. 63d

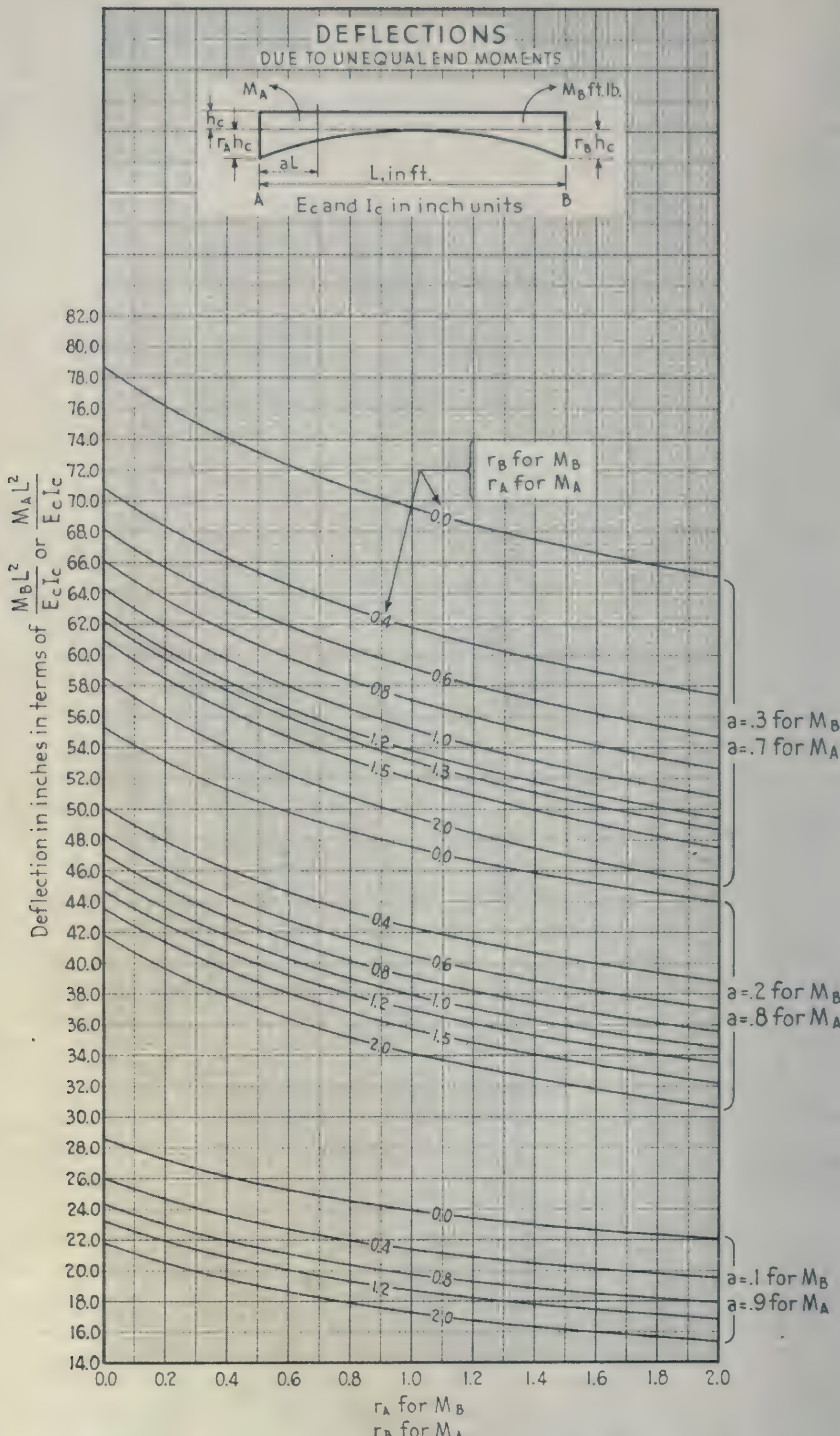

DEFLECTIONS
DUE TO UNEQUAL END MOMENTS
M_A
M_B ft.lb.
h_c
$r_A h_c$
$r_B h_c$
aL
L, in ft.
A
B
E_c and I_c in inch units
r_B for M_B
r_A for M_A
Deflection in inches in terms of $\frac{M_B L^2}{E_c I_c}$ or $\frac{M_A L^2}{E_c I_c}$
a=.3 for M_B
a=.7 for M_A
a=.2 for M_B
a=.8 for M_A
a=.1 for M_B
a=.9 for M_A
r_A for M_B
r_B for M_A

Fig. 63e

vertical deflections of horizontal concrete members increase at a diminishing rate under sustained loads, and it has been shown that ultimate deflections under such loads are approximately three times the original at the end of two years, with very little increase beyond that time*. Consequently it is recommended that approximately one-third the instantaneous secant modulus of elasticity be used in determination of dead load deflections.

EXAMPLE

Determine the dead load deflection curve for the four-span bridge of Design Problem No. 3.

For the purpose of this example a value of $E_c = 1{,}500{,}000$ p.s.i. will be used on the assumption that the actual modulus of elasticity of the concrete used will be about 4,500,000 p.s.i., which is about the value for 3,000-p.s.i. concrete at 28 days**.

From Design Problem No. 3:

$h_c = 30.5$ in. $I_c = 79{,}000$ in.^{4} $r_{AB} = 0$ $r_{BA} = r_{BC} = 1.3$ $r_{CB} = 1.5$
$w = 1{,}248$ lb. $W_{BA} = W_{BC} = 714$ lb. $W_{CB} = 825$ lb.
$M_{BA} = M_{BC} = -856$ ft. kips $M_{CB} = M_{CD} = -980$ ft. kips

Then for Spans 1 and 4

$$\frac{wL^4}{E_cI_c} = \frac{1{,}248 \times 57.5^4}{1.5 \times 10^6 \times 79{,}000} = 0.115$$

$$\frac{W_{BA}L^4}{E_cI_c} = \frac{714 \times 57.5^4}{1.5 \times 10^6 \times 79{,}000} = 0.066$$

$$\frac{M_{BA}L^2}{E_cI_c} = \frac{-856{,}000 \times 57.5^2}{1.5 \times 10^6 \times 79{,}000} = -0.024$$

and for Spans 2 and 3

$$\frac{wL^4}{E_cI_c} = \frac{1{,}248 \times 79.0^4}{1.5 \times 10^6 \times 79{,}000} = 0.410$$

$$\frac{W_{BC}L^4}{E_cI_c} = \frac{714 \times 79.0^4}{1.5 \times 10^6 \times 79{,}000} = 0.234$$

$$\frac{W_{CB}L^4}{E_cI_c} = \frac{825 \times 79.0^4}{1.5 \times 10^6 \times 79{,}000} = 0.271$$

$$\frac{M_{BC}L^2}{E_cI_c} = \frac{-856{,}000 \times 79.0^2}{1.5 \times 10^6 \times 79{,}000} = -0.045$$

$$\frac{M_{CB}L^2}{E_cI_c} = \frac{-980{,}000 \times 79.0^2}{1.5 \times 10^6 \times 79{,}000} = -0.052$$

---

*"Shrinkage and Time Effects in Reinforced Concrete" by F. R. McMillan, *University of Minnesota Studies in Engineering No. 3.* "Flow of Concrete Under Sustained Load" by E. B. Smith, *Proceedings of American Concrete Institute*, Vol. 12, page 317. "Effect of Plastic Flow in Rigid Frames of Reinforced Concrete" by F. E. Richart, R. L. Brown and T. G. Taylor, *Proceedings of American Concrete Institute*, Vol. 30, page 181. *Influence Lines Drawn as Deflection Curves*, published by Portland Cement Association.

**For general design purpose, E_c for this quality concrete is taken as 3,000,000 p.s.i.

TABLE XV—Deflections in Spans 1 and 4*

Point	Uniform load w		Parabolic load W_{BA}		End moment M_{BA}		Total deflections	
	Coefficients[1]	Deflections	Coefficients[2]	Deflections	Coefficients[3]	Deflections	Inches	Feet
.1	6.46	0.74	0.47	0.03	23.0	−0.55	0.22	0.018
.2	12.15	1.40	0.91	0.06	44.3	−1.06	0.40	0.033
.3	16.39	1.89	1.27	0.08	62.2	−1.49	0.48	0.040
.4	19.00	2.19	1.53	0.10	74.8	−1.80	0.49	0.041
.5	19.47	2.24	1.64	0.11	80.6	−1.94	0.41	0.034
.6	17.79	2.05	1.58	0.10	77.8	−1.87	0.28	0.023
.7	14.40	1.66	1.33	0.09	66.4	−1.60	0.15	0.013
.8	9.96	1.15	0.95	0.06	48.1	−1.16	0.05	0.004
.9	5.08	0.59	0.49	0.03	25.3	−0.61	0.01	0.001

[1] See Fig. 59. [2] See Fig. 61. [3] See Fig. 63.

*The structure being symmetrical about the center support, the deflection at any point in Span 4 will be the same as that at the symmetrically opposite point in Span 1.

TABLE XVI—Deflections in Spans 2 and 3†

Pt.	Uniform load w		Parabolic load W_{BC}		Parabolic load W_{CB}		End moment M_{BC}		End moment M_{CB}		Total deflections	
	Coef.[1]	Def.	Coef.[2]	Def.	Coef.[2]	Def.	Coef.[3]	Def.	Coef.[3]	Def.	In.	Ft.
.1	4.39	1.80	0.45	0.11	0.36	0.10	23.4	−1.05	17.4	−0.91	0.05	0.004
.2	8.65	3.55	0.88	0.21	0.71	0.19	44.4	−2.00	34.4	−1.79	0.16	0.013
.3	12.43	5.10	1.22	0.29	1 03	0.28	60.8	−2.74	50.4	−2.62	0.31	0.026
.4	15.16	6.22	1.43	0.33	1.29	0.35	70.5	−3.17	63.3	−3.29	0.44	0.037
.5	16.18	6.64	1.45	0.34	1.43	0.39	71.5	−3.22	70.2	−3.65	0.50	0.042
.6	15.07	6.18	1.30	0.30	1.39	0.38	63.9	−2.88	68.7	−3.57	0.41	0.034
.7	12.27	5.03	1.03	0.24	1.18	0.32	50.6	−2.28	58.7	−3.05	0.26	0.022
.8	8.49	3.48	0.71	0.17	0.84	0.23	34.6	−1.56	42.4	−2.21	0.11	0.009
.9	4.30	1.76	0.35	0.08	0.43	0.12	17.4	−0.78	22.3	−1.16	0.02	0.002

[1] See Fig. 59. [2] See Fig. 61. [3] See Fig. 63.

†See footnote Table XV.

The final deflection curve for Spans 1 and 2 is shown in Fig. 64.

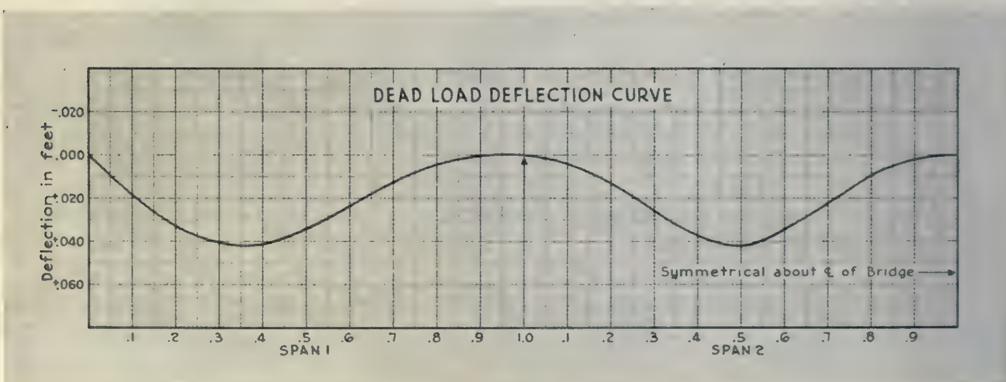


Fig. 64

Section VIII—Four-Span Unsymmetrical Bridges and Bridges Involving Units of Five or More Spans

Final moment formulas for four-span unsymmetrical bridges and for structures involving units of five or more spans, either symmetrical or unsymmetrical, become somewhat cumbersome. Such structures are seldom encountered, but, when necessary, moment coefficients at the supports may be found by distributing an assumed fixed end moment of -100 at each end of each span as illustrated in the following example. Note that it is necessary at each section investigated to add the moments resulting from the distribution of the fixed end moment at each end of the loaded span in order to get the final moments due to the loads within the span. The coefficients thus obtained are applied to the actual fixed end moments for the various positions of loading, thereby giving the final moments at the supports.

EXAMPLE—Four-Span Unsymmetrical Bridge Unit Load in Span *BC*

The deck is assumed to be integral with the piers at *B* and *C* and freely supported at *A*, *D* and *E*. The carry-over and distribution factors have been arbitrarily assumed as indicated for the purpose of the example. It should be noted that the sum of the distribution factors at *B* and *C* do not equal unity since the factor for the pier is not shown. At support *D*, however, the sum of the distribution factors is unity because the deck is freely supported at that point.

A	B	B	C	C	D	D	E	
-.82	-.43	-.73	-.68	-.63	-.75	-.44	-.75	CARRY - OVER FACTORS
0	.30	.29	.30	.32	.49	.51	0	DISTRIBUTION FACTORS
		-100.0						FIXED END MOMENT
	-30.0	+29.0						1ST DISTRIBUTION
			-21.2					1ST CARRY - OVER
			+6.3	-6.8				2ND DISTRIBUTION
		-4.3			+4.3			2ND CARRY - OVER
	-1.3	+1.3			-2.1	+2.2		3RD DISTRIBUTION
			-0.9	+1.6				3RD CARRY - OVER
			+0.7	-0.8				4TH DISTRIBUTION
		-0.5			+0.5			4TH CARRY - OVER
	-0.2	+0.2			-0.2	+0.3		5TH DISTRIBUTION
			-0.1	+0.2				5TH CARRY OVER
			+0.1	-0.1				6TH DISTRIBUTION
	-31.5	-74.3	-15.1	-5.9	+2.5	+2.5		FINAL MOMENT
	.315	.743	.151	.059	-.025	-.025		COEFFICIENTS = (FINAL MOMENTS) / -100

By the same procedure and $M^F_{CB} = -100$, the following coefficients may be obtained:

$$.068|.161 \quad .714|.279 - .117| - .117$$

The final moment at supports due to a load in BC will be:

$M_{AB} = 0$

$M_{BA} = 0.315\ M^F_{BC} + 0.068\ M^F_{CB}$

$M_{BC} = 0.743\ M^F_{BC} + 0.161\ M^F_{CB}$

$M_{CB} = 0.151\ M^F_{BC} + 0.714\ M^F_{CB}$

$M_{CD} = \ \ 0.059\ M^F_{BC} + 0.279\ M^F_{CB}$

$M_{DC} = -0.025\ M^F_{BC} - 0.117\ M^F_{CB}$

$M_{DE} = -0.025\ M^F_{BC} - 0.117\ M^F_{CB}$

$M_{ED} = 0$

Similarly, coefficients and expressions for final moments at each support due to loads in each span may be obtained.

The moments in the integral piers are obtained as the difference between moments in the deck on each side of the support and act in the same direction as the smaller deck moment. For the loading in this problem, the moment at top of pier B causes tension on the outside of the pier and equals:

$(0.743M^F_{BC} + 0.161M^F_{CB}) - (0.315M^F_{BC} + 0.068^F_{CB}) = (0.428M^F_{BC} + 0.093M^F_{CB})$

Section IX—Skew Bridges

The stresses in a skew slab* differ materially from those in a straight slab; this difference increases with the angle of skew. Loads travel to the supports in proportion to the rigidity of the various possible paths; therefore, the main portion of any load tends to reach the supports in a direction normal to faces of abutments and piers. This means that planes of maximum stresses are not parallel to the centerline of roadway, and that the deflection of such slabs produces a warped surface.

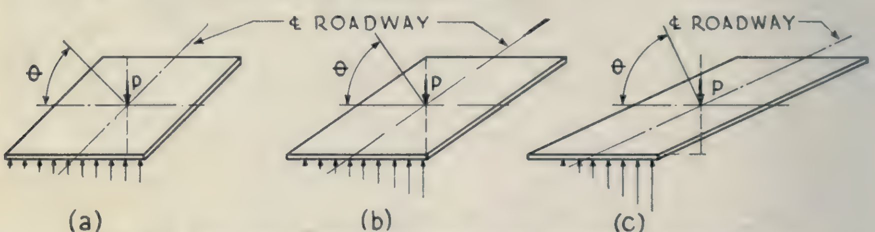


Fig. 65. Reactions on abutments vary with increase of skew angle.

Fig. 65a, b and c show schemetically how the reactions change with the increase of skew angle. This differential in support reaction exists only at the ends of freely supported slabs; in continuous slabs, the loads coming from adjacent spans, to a large extent, equalize the pier reactions. The exact variation of reactions for different types of loading and different skew angles is not known. It has been shown, however, that the reaction of an abutment of a 60 deg. skew arch uniformly loaded varied from zero to twice average pressure**. This fact should be kept in mind when designing footings and estimating required pile capacities.

In the absence of more exact methods of analysis, the following is recommended for the design of slab spans***:

For skews up to 20 deg., use span along centerline of roadway; design slab as straight, and assume footing reactions at obtuse angle corners at free ends to increase from 0 to 50 per cent above the average pressure according to amount of skew.

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*University of Illinois, Engineering Experiment Station, Bulletin 332.

**See *Public Roads*, November, 1925.

***See "Approximate Design Method for Concrete Skew Rigid Frames" by E. F. Gifford, *Engineering News-Record*, May 3, 1934, page 574.

For skews from 20 deg. to 50 deg. use span perpendicular to supports; obtain thickness of slab and amount of steel as though the slab were straight, then multiply steel required by secant squared of the skew angle, if steel is placed parallel to centerline of roadway. Assume footing reaction at obtuse angle corner to increase from 50 per cent to 90 per cent above the average pressure at the freely supported ends.

For skews larger than 50 deg., a T-girder bridge should be used even though the spans are short.

When T-girders are used, the footing reactions at the obtuse angle corners are somewhat greater than for a straight bridge, but the increase is small compared to that of a slab, and is usually ignored.

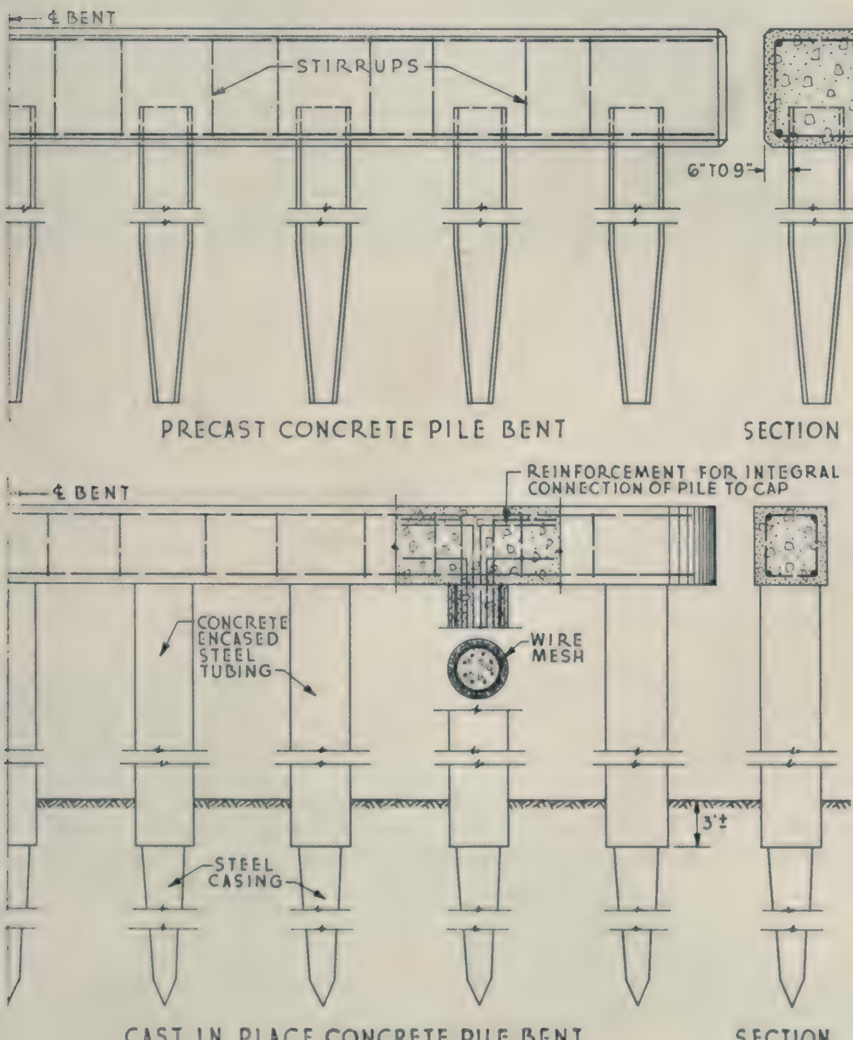

Fig. 66

Section X—Substructures

Pile Bents

Pile bents now commonly used for piers and abutments consist of precast and cast-in-place piles, capped with a beam. The precast type is the more commonly used. The piles are spaced so as to utilize the maximum bearing capacity as nearly as practicable, with a minimum spacing of about 4-ft. centers. The usual sizes for highway bridges are from 12 to 20 in. in minimum transverse dimension, either square, round, or hexagonal; although

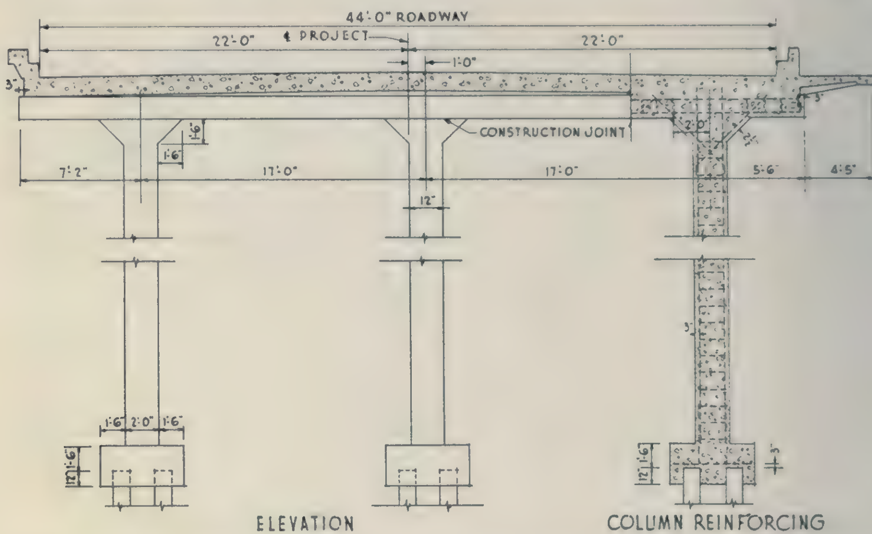

Fig. 67 (*above and below*). Open frame bents spaced nominally 20 ft. apart with individual footings carried on piles were used for the substructure of the continuous slab Wichita Viaduct. Designed by the Kansas State Highway Commission; George W. Lamb, bridge engineer.

Fig. 68. Rail-highway grade separation over L. & N. Railway, Clark County, Ky., shows open frame bents with individual footings set in rock. Designed by Kentucky State Highway Dept.; H. R. Creal, bridge engineer.

sizes up to 24 in. solid and 30 in. hollow are being used for railroad bridges.

When the tops of the piles are encased by the cap beam without first stripping away the concrete from the reinforcement the cap should project beyond the pile from 6 to 9 in., as shown in Fig. 66. If the piles are cut off from 18 to 24 in. above the required elevation of the bottom of the cap and the concrete is stripped from around the longitudinal steel to provide bond with the cast-in-place beam, the cap need be little or no wider than the piles. The saving in cap concrete will offset the cost of stripping the concrete from the pile heads, and permit the use of a neater appearing cap. The cap must be designed for moments and shears if the deck loads do not fall directly over the piles. Typical reinforcement in caps is shown in Fig. 66.

Cast-in-place concrete piles using steel casings can be driven with lighter equipment than can the precast piles. Piles having casings strong enough to be driven without a core or mandrel need no reinforcement other than the casing itself. This type of pile permits easy integral construction of cap beam and pile core; it is only necessary to insert the required dowel bars in the core after being filled. It is desirable to place a concrete shell, reinforced with wire mesh, around the steel casings from below ground to bottom of cap beam as a protection against corrosion and for the sake of producing a unified appearance. The costs of the two types of piles in place are approximately the same when the equipment required to drive each type is equally available.

Fig. 69. Continuous slab bridge, Glenn County, Calif., shows pile caps integral with deck and the use of slope walls. Designed by California State Division of Highways; F. W. Panhorst, bridge engineer.

Open Frame Bents

Open frame bents commonly used consist of a spread footing, bottom beam, top beam and square posts spaced 8 to 16-ft. centers. When posts are spaced 12 ft. or more apart, it may prove more economical to provide individual footings instead of a bottom beam, see Fig. 67. When footings are on rock, the posts should be set into the rock far enough to secure a level bearing on sound rock and have only a small bulb-like base around the posts as in Fig. 68.

When the loads carried by the posts do not require larger sizes when designed as columns, the sides of the square posts may be taken as 1 in. per foot of height between beams, about 20 in. being the minimum size suitable for convenient construction.

The top beam may be made integral with the deck if desired, see Figs.

Fig. 70. Continuous reinforced concrete girder bridge, spans approximately 83, 111, 111 and 83 ft., near Kenney, Ill. Designed by Illinois Division of Highways; George F. Burch, engineer of bridges.

Fig. 71. Continuous concrete girder bridge over Los Angeles River, 64-ft. spans, with piers built integral with deck. Designed by California State Division of Highways; F. W. Panhorst, bridge engineer.

67 and 69. If this is done some reduction in size of posts is desirable in order to keep the stresses due to change in deck length as low as possible, even though this may make construction a little more difficult. However, an 18-in. square column should be about the practical minimum.

Solid Piers

When width of a solid pier is not determined by size of bearing areas, or mass to resist ice jams or floating debris, the width of pier, just under the cap, if cap is used, may be taken as 1 in. per foot of height between footing and cap, with a practical minimum of about 18 in. It is desirable to provide a batter of at least 0.2 in. per foot of height on each side of high piers. If high-water elevation is at some distance from top of pier, it may prove the more economical to make piers solid only up to high-water elevation as shown in Fig. 70. If pier is built integral with deck as shown in Fig. 71, the minimum thickness should be reduced to 14 in. for low piers in order to keep stresses due to change in deck length as low as possible.

Open Abutments

The open type abutment, either pile bent or framed bent, presents several advantages which merit careful consideration; such as:

1. A bridge with open type abutments, although longer, costs less than one with closed abutments, if the abutments are to be much over 10 ft. high.
2. Experience has shown maintenance charges on the open abutments when properly constructed to be less than on the closed type.
3. The over-all length of a bridge with open abutments being longer than one with closed abutments, the height of fill at abutments is reduced and the amount of roadway pavement is also reduced.

Fig. 72. Three-span bridge near Milford, Ill., shows use of pile bent piers and open pile abutments. Designed by Illinois Division of Highways; George F. Burch, engineer of bridges.

4. The channel section with its sloping ends, see Fig. 69, provides a more efficient waterway at flood stage than does the rectangular section of the closed abutments.

When pile bents are used as open abutments, see Fig. 73, the bent should be set back of theoretical position about 0.4 in. plus 0.04 in. per foot of fill to offset movement when backfill and approach slab are placed.

When open frame bents are used as open abutments they should also be set back or built tilted back from the final position an amount equal to 0.04 in. per foot of height from bottom of footing to crown of roadway to take care of probable forward movement.

Closed abutments will generally be used in urban communities and where site conditions preclude the longer bridge required with open abutments to avoid encroachment of the embankments on the waterway or

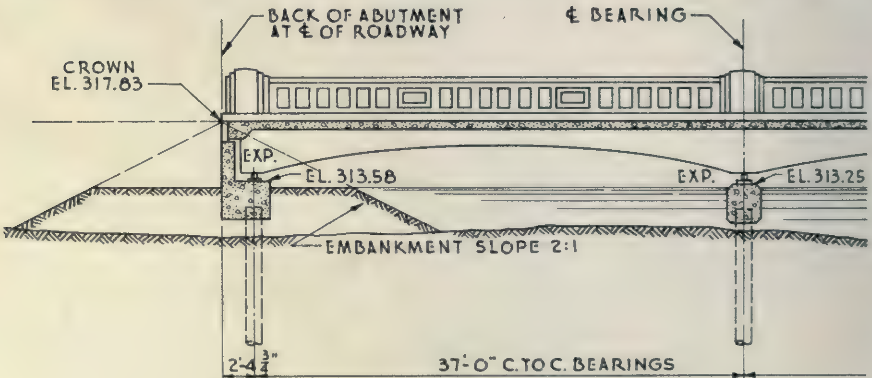

Fig. 73. Pile bent piers and open abutment.

Fig. 74. The separation of wingwalls from breastwalls is illustrated in the continuous girder bridge near Villa Park, Ill. Designed by Illinois Division of Highways; George F. Burch, engineer of bridges.

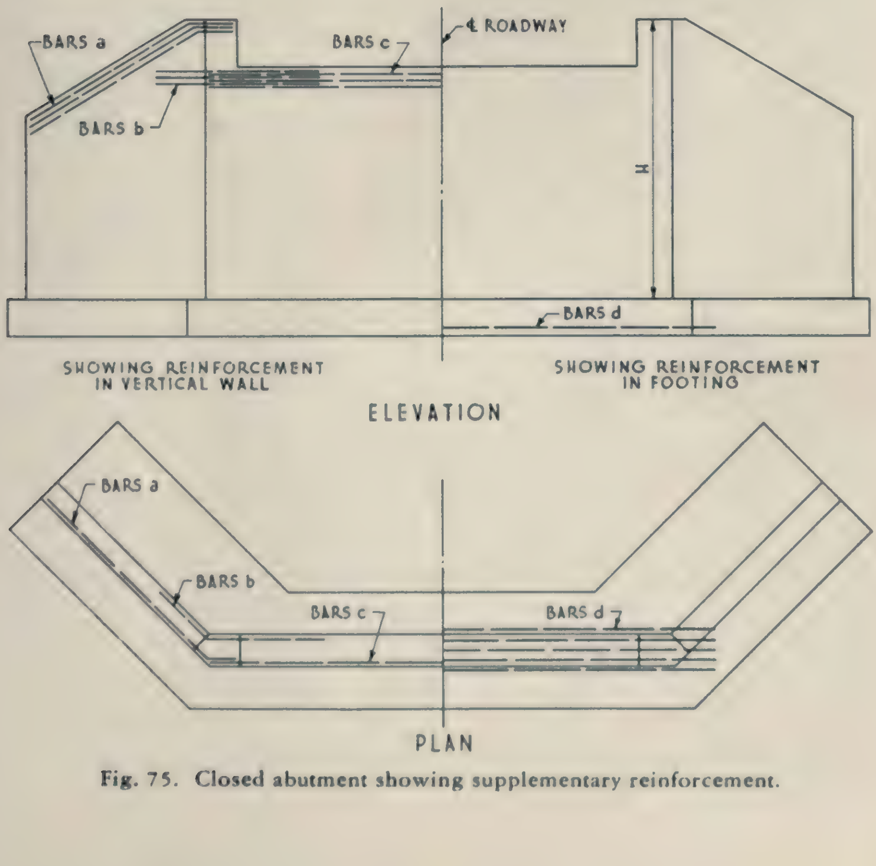

Fig. 75. Closed abutment showing supplementary reinforcement.

bridge opening. Closed abutments act as retaining walls and at the same time transmit the superstructure load to the foundation. This combination of functions together with unequal shrinkage of exposed and unexposed parts, earth pressure on wingwalls, and other factors must be carefully considered in design*. It is desirable to separate wingwalls from breastwalls, see Fig. 74, but when this is not done special precaution should be taken to

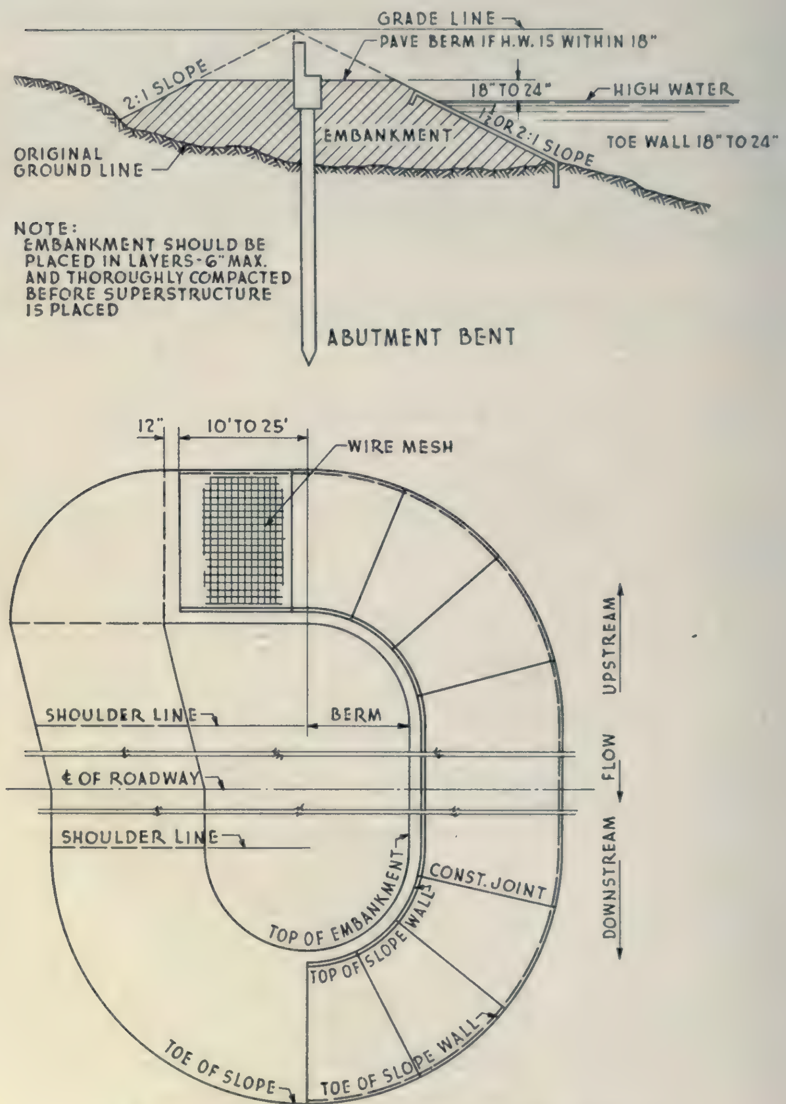

Fig. 76. Concrete slope wall used at open abutments.

*See *Concrete Bridge Details*, supplied free in United States and Canada upon request to Portland Cement Association.

The continuous girder bridge over Wabash River, Lockport, Ind., consists of four 95-ft., two 75-ft., and two 50-ft. spans. Designed by C. R. McAnlis, consulting engineer, Fort Wayne, Ind.

reinforce the structure where experience has shown cracks are likely to occur when only the usual abutment reinforcement is provided. Fig. 75 shows a common type of closed abutment with supplementary reinforcement provided where experience shows it is necessary. Bars marked *a* to *d* are in addition to the reinforcement provided to resist the usual design loads.

Just as in the case of open abutments, there is a tendency of closed abutments to tilt forward due to the backfill pressure but it is even more pronounced. This tendency can be overcome by setting the wall back of the final position, if battered on the front face; or giving it an initial tilt backward, if intended to be vertical, an amount equal to 0.04 in. per foot of height.

Slope Walls

At any abutment where an appreciable contraction in width of flood plain is made, erosion may be expected, unless movement of flood water in the overflow area is slow and subsidence is also slow. Fig. 76 shows a plan and section for 4-in. thick concrete slope wall at an open abutment. Fig. 69 shows such a slope wall in service.

At closed abutments restricting the width of flood plain is more serious than at open abutments; so in most cases the embankment slope just behind an abutment is subject to erosion during floods, and if not paved for a few feet or protected with a gutter take-off along side of approach pavement, erosion just behind the wings is almost certain and the embankment must be maintained after each major flood or progressive failure is likely.

Section XI—Details

Construction Joints

(1) *Horizontal construction* joints should be made at:

(a) Tops of curbs in continuous slab bridges;

(b) Tops of slabs in continuous T-girder bridges.

Curbs of slab bridges participate in carrying "edge moments" under most design specifications and therefore should be cast integral with the deck slab. Curbs on continuous T-girder bridges are not considered structural members; hence it is desirable to have a construction joint between the slab and the curb and the latter should be placed after the centering has been struck to prevent participation of the curb in carrying the dead load.

Emergency construction joints in the deck should be vertical. These joints are discussed under "Vertical Joints."

If the size of an abutment precludes the possibility of placing the concrete in a continuous operation, it is preferable to use vertical rather than horizontal construction joints. Sometimes, however, horizontal joints cannot be avoided. In such cases special care must be exercised to make a straight, flush and watertight joint*.

(2) *Vertical construction* joints should be placed, when necessary:

(a) At or near points of contraflexure in beams, slabs and T-girders;

(b) At points of high positive moments, only if unavoidable, and if some additional tension steel is used or exceptional care is taken in bonding new to old concrete;

(c) In wingwalls of abutments as near as practical to the front wall and at about 20-ft. intervals, and in front walls if longer than 30 ft.

If, in case of an emergency, construction must be stopped in a slab, beam or girder at a point of high negative moment stress, special care *must* be taken in bonding the new concrete to the old**.

Vertical joints in abutments are likely to open and therefore should be provided with waterstops. Joints in the wings, separating them from the front wall, logically separate two members having very different functions and types of stress.

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*For information, see *Construction Joints*, supplied free in United States and Canada upon request to Portland Cement Association.

**See "Bonding New Concrete to Old", *Proceedings of American Concrete Institute*, Vol. 30, page 422. The most efficient method showed as high as 96 per cent efficiency. This method is approximately as follows:

After old concrete is placed, clean joint surface with a pressure water jet just before final set—about 6 hours; cover joint and keep it wet until new concrete is placed; flush surface with 1:2 portland cement mortar just before new concrete is placed; then vibrate concrete into place; cover and keep moist for several days

It was found that there was only a little loss in efficiency in delaying placing new concrete up to 20 days.

Expansion and Contraction Joints

Careful consideration of such details as expansion and contraction joints is as important as stress analysis. Experience has shown that poor joint details result in high maintenance and even serious injury to structures.

Adequate joints between deck and wingwalls or parapets have not usually been provided of sufficient width for wide roadways with the result that lateral expansion of the deck has caused serious cracking as illustrated in the sketch, Fig. 6 in booklet, *Concrete Bridge Details**, page 11.

Expansion joints should be placed:

- (a) At free ends of all continuous units to provide for longitudinal expansion;
- (b) Between deck and wing or parapet walls to take care of transverse expansion.

Joints smaller than 2 in. should be so detailed that no water, dirt or trash passes through them. The width of joint should be four times that actually required and should be filled with good grade mastic and should have a copper waterstop. See Fig. 8 in concrete information sheet, *Expansion Joints in Concrete Bridge Decks and Retaining Walls**.

Joints larger than 2 in. allowing free passage of water should be provided with a gutter to carry off water. See Fig. 4 in the above-mentioned concrete information sheet for a gutter detail. Gutter outlet should carry water off beyond supporting pier abutment.

Contraction joints should be placed in members with little reinforcement in the direction of restraint; for example, a longitudinal joint on centerline of a 40-ft. roadway. Such a joint also makes for easier construction.

Joints in Handrails

Joints in handrails are necessary to secure discontinuity of rails so they will not participate with the deck slab or girders in carrying any of the live or dead load. Where the handrail and curb of a girder bridge are combined and the rail joints extend through the curb to the top of the slab, the joints in the curb should be calked with a good mastic.

Open joints in handrails should be placed as follows:

- (a) ½-in. joints at 8 to 10-ft. centers in positive moment regions;
- (b) ¼-in. joints at 4 to 6-ft. centers in negative moment regions;
- (c) ¼-in. joints on each side of rail posts over piers.

Diaphragms between Girders

No diaphragms are really needed between girders of continuous T-girder bridges to stiffen the girders for the reasons that only short lengths of the girders on each side of the piers are subjected to high compressive stresses; the stems are restrained at the top by the integral slab and at the bottom by friction on the bearings. The beams, being continuous, are restrained at the points of contraflexure by the longitudinal tension in beams between points of contraflexure.

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*Supplied free in U. S. and Canada upon request to the Portland Cement Association.

Diaphragms extending three-fourths down from slab to bottom of girder and 6 to 10 in. in width may be desirable at supports for construction purposes, and safety against a possible sliding on the bearings due to transverse expansion or contraction. They also serve the very useful purpose of preventing deflection of slab between girders due to longitudinal tension over supports. Diaphragms at any other points are a direct loss in economy, unless small diaphragms are used for wide girder spacing so that roadway slab can be designed as partially fixed on four sides.

Drainage

Under "Expansion and Contraction Joints", the importance of carrying drainage beyond supports was pointed out. It is likewise very important to provide sufficient drains and outlets to prevent any appreciable accumulation of water at expansion devices or at ends of bridge*. Special care should be taken in providing and detailing drainage facilities at abutments; this is as important for the closed type abutments as for the open type. If adequate drainage facilities are not provided, the water passing drains on the bridge will run off around the wingwalls of a closed abutment, carrying away the fill progressively and finally will undermine the approach slab. At open abutments the water likewise will run off around the end of the parapet walls and if the parapet does not extend out and down far enough to protect the berm, the water will cross over under the bridge and seriously erode the berm.

These undesirable conditions can be almost wholly eliminated by providing a gutter section along the sides of the approach roadway slab which will carry the water down the road into a gutter outlet, or by constructing a suitable catch basin, either rigidly attached to the back of the abutment or placed far enough behind the abutment so that a settlement of the basin will not permit water to get in behind the abutment.

Bearings

The type of bearings used for concrete bridges will depend upon amount of movement of the bridge at the bearings and upon the loads carried by the individual beams. Bearing plates are satisfactory where the pressures are low, say not more than 400 lb. per sq.in. of bearing surface, and where the maximum movement is large in respect to bearing pressures.

The bottom bearing plate should rest on a sheet lead plate, say $\frac{1}{8}$ in. thick; plates should be made of high corrosive-resisting steel, such as phosphor-bronze, complying with Specification 64 of Society of Automotive Engineers. The top plate should be separated from the bottom plate by a good, reliable lubricant such as a $\frac{1}{16}$-in. sheet of asbestos fibers cemented together with a medium, rendering it tough and pliable; and the underside of the top plate should be well coated with graphite.

The lead plate will also serve to equalize bearing pressures on the base when the span deflects under live load, and thus prevent high edge pressures.

Where bearing reactions are high, and bearing plates are used at fixed supports, the lower one should be ground to a large radius, so that a line

*See chapter on "Drainage" in *Concrete Bridge Details*, published by the Portland Cement Association. Sent free in United States and Canada upon request.

bearing is maintained under unbalanced live loading. At expansion bearings subjected to heavy loads, solid steel rollers are very satisfactory in sizes from 3 to 10 in. in diameter; when larger rollers are required, a segmental or built-up type* may prove the cheaper.

If a collar is specified on the roller type, clearance should be provided between top plate and collar for possible transverse contraction and expansion of deck. A stop should be provided on the bottom plate so that rollers will not roll off the plate during construction.

Bearings should be inspected periodically, cleaned of dirt or debris, and realigned if necessary. This is particularly advisable after flood waters have been up around them.

Falsework and Removal of Forms

The quality of falsework required for continuous concrete bridges does not differ from that for any reinforced concrete bridge. It is, however, important that the falsework be as strong and as rigid as economically practical in order to insure that proper lines and grade of structure are obtained. Good appearance will allow little deviation from theoretical soffit and water table lines. Falsework may be so constructed that wedging during the placing of concrete will insure almost perfect lines, and falsework should be so constructed that the main supporting members may be left in place after the siding and flooring have been removed.

The supporting falsework members should not be removed from any span of a continuous unit until the concrete has attained a modulus of rupture equal to *at least* 50 per cent of compressive stress used in design; nor should they be removed under the deck during the curing period.

These requirements must be rigidly complied with to insure avoidance of cracks over intermediate supports at working loads.

Curing

The decks of continuous concrete bridges should be kept thoroughly wet not less than 10 days. Falsework supports should not be removed during the curing period.

Wearing Surfaces

Wearing surfaces on concrete bridges today are considered unnecessary and undesirable. The wear produced by modern traffic is insignificant, and the reduction in strength on this account is only a small fraction of the increase in strength of concrete after a bridge is put in service. Construction technique and equipment make it just as easy, if not easier, to get a smooth surface on the structural slab when placed as it is to get a good finish when the wearing surface is placed later. The omission of a wearing surface eliminates a second curing period. Furthermore, wearing surfaces will separate from the slab unless special care is taken to secure a good bond to the structural slab. Water will then get under the wearing surface and

*See standard roller design of the U. S. Bureau of Roads. This is made from H-sections.

freeze, causing troublesome maintenance, if not permanent injury to the bridge. Wearing surfaces are expensive; their first cost may be as much as 5 per cent of the total cost of the bridge.

An additional problem is presented by a separate wearing course in the design of continuous bridges because of the uncertainty of their interaction with the slab in carrying live load when placed after removal of falsework, and in carrying both dead and live load if placed before falsework is removed. In either case the stiffness of the deck is changed an indeterminate amount, so if any provision is made for surface wear it should be made integral with the slab, and the increased slab thickness should be used in finding stiffness and moments of inertia used for finding moments and shears.

Tension Reinforcement over Supports

In order to simplify construction and facilitate placing concrete in the stem, and to insure integrity of the slab over intermediate supports of continuous T-girder bridges, the girder reinforcement should be spread out into the slab. Briefly listed, the chief advantages and reasons for distributing the reinforcement are:

(1) Tension stresses in slab are practically the same across the whole breadth of slab at maximum loading.
(2) If reinforcement is not so placed, cracks occur between stems at sections of high negative moments, and large cracks are particularly undesirable on the top of the slab.
(3) This arrangement gives some economy in amount of tension reinforcement.
(4) Spreading out of steel enables easy placement of concrete in the deep T-girders.
(5) This arrangement makes possible use of W or multiple-leg stirrups which will not interfere with placing main longitudinal tension bars.

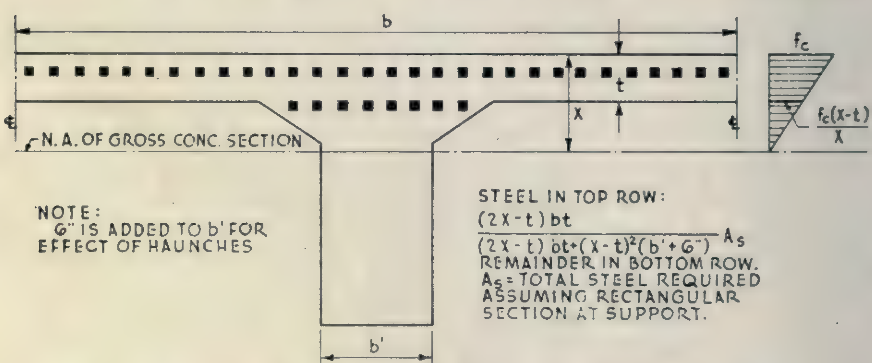

Fig. 77. Suggested distribution of girder reinforcement over supports.

Fig. 77 shows a method of determining the quantity of reinforcement in each layer, the top one of which extends from centerline to centerline between girders.

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